4.1 The Percent Equation
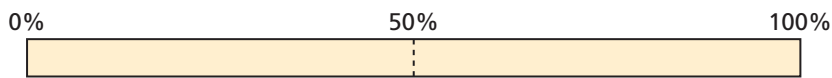
**Essential Question** How can you use models to estimate percent questions?

1 ACTIVITY: Estimating a Percent

Work with a partner. Estimate the locations of 50%, 75%, 40%, 6%, and 65% on the model. 50% is done for you.

0%	50%	100%

2 ACTIVITY: Estimating a Part of a Number

The statement "25% of 12 is 3" has three numbers. In real-life problems, any one of these numbers can be unknown.

Part
Whole
$$\frac{3}{12} = 0.25 = 25\%$$
Percent
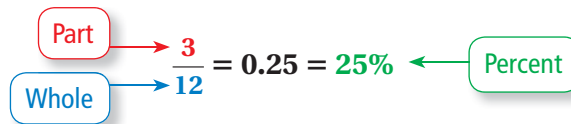
Which number is missing?	Question	Type of Question
3	What is 25% of 12?	Find a part of a number.
25%	3 is what percent of 12?	Find a percent.
12	3 is 25% of what?	Find the whole.

Work with a partner. Estimate the answer to each question using a model.

a. **Sample:** What number is 50% of 30?
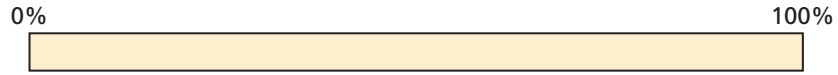
0%	50%	100%
0	15	30
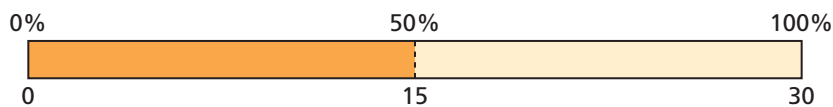
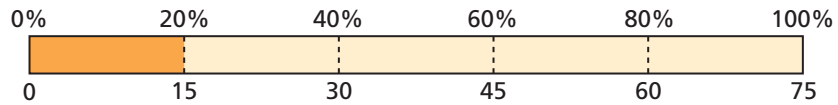⁚⁚ So, from the model, 15 is 50% of 30.

b. What number is 75% of 30? **c.** What number is 40% of 30?

d. What number is 6% of 30? **e.** What number is 65% of 30?

3 ACTIVITY: Estimating a Percent

Work with a partner. Estimate the answer to the question using a model.

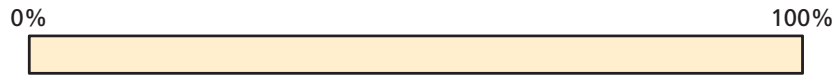0%   100%
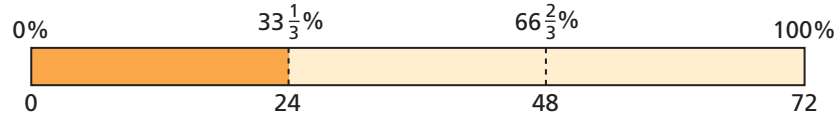
a. Sample: 15 is what percent of 75?

0%	20%	40%	60%	80%	100%
0	15	30	45	60	75

So, 15 is 20% of 75.

b. 5 is what percent of 20?   **c.** 18 is what percent of 40?

d. 50 is what percent of 80?   **e.** 75 is what percent of 50?

4 ACTIVITY: Estimating a Whole

Work with a partner. Estimate the answer to the question using a model.

0%   100%

a. Sample: 24 is $33\frac{1}{3}$% of what number?

0%	$33\frac{1}{3}$%	$66\frac{2}{3}$%	100%
0	24	48	72

So, 24 is $33\frac{1}{3}$% of 72.

b. 13 is 25% of what number?   **c.** 110 is 20% of what number?

d. 75 is 75% of what number?   **e.** 81 is 45% of what number?

What Is Your Answer?

5. IN YOUR OWN WORDS How can you use models to estimate percent questions? Give examples to support your answer.

Practice

Use what you learned about estimating percent questions to complete Exercises 4–9 on page 162.

Key Vocabulary
percent, p. 160

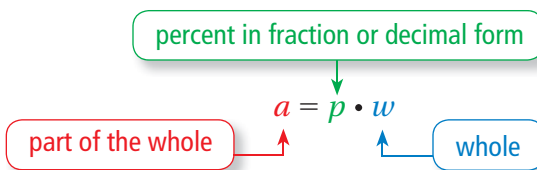A **percent** is a ratio whose denominator is 100. Here are two examples.

$$4\% = \frac{4}{100} = 0.04 \qquad\qquad 25\% = \frac{25}{100} = 0.25$$

Key Idea

The Percent Equation

Words   To represent "a is p percent of w," use an equation.

percent in fraction or decimal form

$$a = p \cdot w$$

part of the whole   whole

Numbers $15 = 0.5 \cdot 30$

EXAMPLE 1   **Finding a Part of a Number**

What number is 24% of 50?   **Estimate**

0%   25%   100%

0   12.5   50

Common Error

Remember to convert a percent to a fraction or decimal before using the percent equation. For Example 1, write 24% as $\frac{24}{100}$.
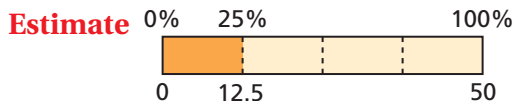
$a = p \cdot w$ Write percent equation.

$= \frac{24}{100} \cdot 50$ Substitute $\frac{24}{100}$ for p and 50 for w.
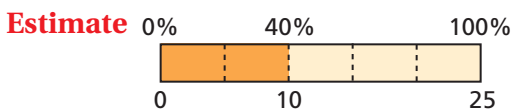
$= 12$ Multiply.

So, 12 is 24% of 50.   **Reasonable?** $12 \approx 12.5$ ✓

EXAMPLE 2   **Finding a Percent**

9.5 is what percent of 25?   **Estimate**

0%   40%   100%

0   10   25

$a = p \cdot w$ Write percent equation.

$9.5 = p \cdot 25$ Substitute 9.5 for a and 25 for w.

$0.38 = p$ Divide each side by 25.

Because 0.38 equals 38%, 9.5 is 38% of 25.   **Reasonable?** $38\% \approx 40\%$ ✓

Multi-Language Glossary at BigIdeasMath.com.

EXAMPLE **3** **Finding a Whole**

39 is 52% of what number?   **Estimate**   0%   50%   100%

0   39   78

$$a = p \cdot w$$ Write percent equation.

$$39 = 0.52 \cdot w$$ Substitute 39 for a and 0.52 for p.

$$75 = w$$ Divide each side by 0.52.

So, 39 is 52% of 75.   **Reasonable?** $75 \approx 78$ ✓

On Your Own

Now You're Ready
Exercises 10–17

Write and solve an equation to answer the question.

1. What number is 10% of 20?   **2.** What number is 150% of 40?

3. 3 is what percent of 600?   **4.** 18 is what percent of 20?

5. 8 is 80% of what number?   **6.** 90 is 18% of what number?

EXAMPLE **4** **Real-Life Application**

a. Find the percent of sales tax on the food total.

b. Find the amount of a 16% tip on the food total.

8th Street Cafe
Fort Lauderdale, FL

DATE: MAY04'10 05:45PM
TABLE: 29
SERVER: CHARITY

Food Total   27.50
Tax   1.65
Subtotal   29.15

TIP: _____

TOTAL: _____

Thank You

a. Answer the question: $1.65 is what percent of $27.50?

$$a = p \cdot w$$ Write percent equation.

$$1.65 = p \cdot 27.50$$ Substitute 1.65 for a and 27.50 for w.
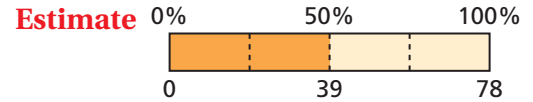
$$0.06 = p$$ Divide each side by 27.50.

Because 0.06 equals 6%, the percent of sales tax is 6%.

b. Answer the question: What tip amount is 16% of $27.50?

$$a = p \cdot w$$ Write percent equation.

$$= 0.16 \cdot 27.50$$ Substitute 0.16 for p and 27.50 for w.

$$= 4.40$$ Multiply.

So, the amount of the tip is $4.40.

On Your Own

7. **WHAT IF?** In Example 4, find the amount of a 20% tip on the food total.

✓ Vocabulary and Concept Check

1. **VOCABULARY** Write the percent equation in words.

2. **REASONING** A number n is 150% of number m. Is n *greater than*, *less than*, or *equal to m*? Explain your reasoning.

3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What number is 20% of 55?	55 is 20% of what number?
20% of 55 is what number?	0.2 • 55 is what number?

✐ Practice and Problem Solving

Estimate the answer to the question using a model.

4. What number is 24% of 80?   5. 15 is what percent of 40?

6. 15 is 30% of what number?   7. What number is 120% of 70?

8. 20 is what percent of 52?   9. 48 is 75% of what number?

Write and solve an equation to answer the question.

① 10. 20% of 150 is what number?   11. 45 is what percent of 60?

② 12. 35% of what number is 35?   13. 32% of 25 is what number?

③ 14. 29 is what percent of 20?   15. 0.5% of what number is 12?

16. What percent of 300 is 51?   17. 120% of what number is 102?

ERROR ANALYSIS Describe and correct the error in using the percent equation.

18. What number is 35% of 20?   19. 30 is 60% of what number?

✗
$a = p \cdot w$
$\quad = 35 \cdot 20$
$\quad = 700$

✗
$a = p \cdot w$
$\quad = 0.6 \cdot 30$
$\quad = 18$

20. **BASEBALL** A pitcher throws 75 pitches. Of these, 72% were strikes. How many strikes did the pitcher throw?

21. **FUNDRAISING** Your school raised 125% of its fundraising goal. The school raised $6750. What was the goal?

22. **SURFBOARD** The sales tax on a surfboard is $12. What is the percent of sales tax?

SALE
$240

PUZZLE There were w signers of the Declaration of Independence. The youngest was Edward Rutledge, who was x years old. The oldest was Benjamin Franklin, who was y years old.

23. x is 25% of 104. What was Rutledge's age?

24. 7 is 10% of y. What was Franklin's age?

25. w is 80% of y. How many signers were there?

26. y is what percent of $(w + y - x)$?

Favorite Sport

Other

40.0%

37.5%
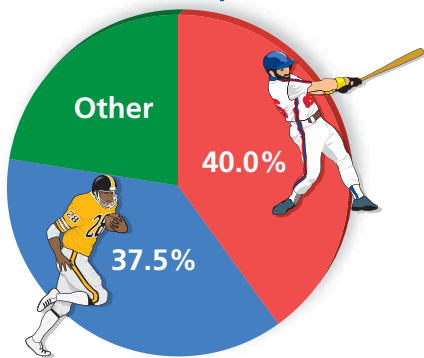
27. REASONING How can you tell whether the percent of a number will be *greater than*, *less than*, or *equal to* the number?

28. SURVEY In a survey, a group of students were asked their favorite sport. "Other" sports were chosen by 18 people.

   a. How many students participated?

   b. How many chose football?

29. WATER TANK Water tank A has a capacity of 550 gallons and is 66% full. Water tank B is 53% full. The ratio of the capacity of tank A to tank B is $11:15$.

   a. How much water is in tank A?

   b. What is the capacity of tank B?

   c. How much water is in tank B?

30. TRUE OR FALSE? Tell whether the statement is *true* or *false*. Explain your reasoning.

   If W is 25% of Z, then $Z:W$ is $75:25$.

31. **Reasoning** The table shows your test results for math class. What test score is needed on the last exam to earn 90% of the total points?

Test Score	Point Value
83%	100
91.6%	250
88%	150
?	300

Fair Game Review What you learned in previous grades & lessons

Simplify. Write as a decimal.

32. $\dfrac{10 - 4}{10}$ **33.** $\dfrac{25 - 3}{25}$ **34.** $\dfrac{105 - 84}{84}$ **35.** $\dfrac{170 - 125}{125}$

36. MULTIPLE CHOICE There are 160 people in a grade. The ratio of boys to girls is 3 to 5. Which proportion can you use to find the number x of boys?

   (A) $\dfrac{3}{8} = \dfrac{x}{160}$ **(B)** $\dfrac{3}{5} = \dfrac{x}{160}$ **(C)** $\dfrac{5}{8} = \dfrac{x}{160}$ **(D)** $\dfrac{3}{5} = \dfrac{160}{x}$