English Spanish

2.5 Systems of Linear Equations

Essential Question How can you solve a system of linear equations?

1 ACTIVITY: Writing a System of Linear Equations

Work with a partner.

Your family starts a bed-and-breakfast in your home. You spend $500 fixing up a bedroom to rent. Your cost for food and utilities is $10 per night. Your family charges $60 per night to rent the bedroom.

a. Write an equation that represents your costs.

Cost, C (in dollars)	=	\$10 per night	•	Number of nights, x	+	\$500

b. Write an equation that represents your revenue (income).

Revenue, R (in dollars)	=	\$60 per night	•	Number of nights, x

c. A set of two (or more) linear equations is called a **system of linear equations**. Write the system of linear equations for this problem.

2 ACTIVITY: Using a Table to Solve a System

Use the cost and revenue equations from Activity 1 to find how many nights you need to rent the bedroom before you recover the cost of fixing up the bedroom. This is the *break-even point* for your business.

a. Copy and complete the table.

x	0	1	2	3	4	5	6	7	8	9	10	11
C												
R												

b. How many nights do you need to rent the bedroom before you break even?

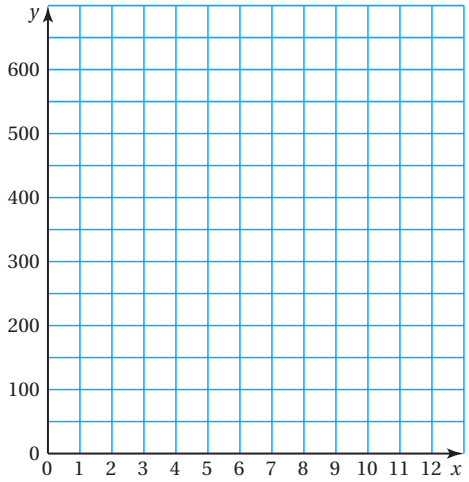
English Spanish

3 ACTIVITY: Using a Graph to Solve a System

a. Graph the cost equation from Activity 1.

b. In the same coordinate plane, graph the revenue equation from Activity 1.

c. Find the point of intersection of the two graphs. The x-value of this point is the number of nights you need to rent the bedroom to break even.



4 ACTIVITY: Using an Equation to Solve a System

a. Write the cost equation from Activity 1.

$C =$

b. Write the revenue equation from Activity 1.

$R =$

c. The break-even point occurs when $C = R$. Set the expression for C equal to the expression for R. You should obtain an equation with x on both sides. Solve this equation for x. The solution is your break-even point.

d. Did you obtain the same break-even point in Activities 2, 3, and 4? If not, check your work. The break-even point should be the same in all three activities.

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you solve a system of linear equations?

6. When solving a system of linear equations, explain why it is a good idea to use two different ways to find the solution.

Practice Use what you learned about systems of linear equations to complete Exercises 3 and 4 on page 80.

English Spanish

2.5 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
system of linear equations, *p. 78*
solution of a system of linear equations, *p. 78*

A **system of linear equations** is a set of two or more linear equations in the same variables. A **solution of a system of linear equations** in two variables is an ordered pair that makes each equation true.

EXAMPLE 1 Solving a System of Linear Equations Using a Table

Solve the system. $y = x - 5$ Equation 1
$y = -3x + 7$ Equation 2

Reading
A system of linear equations is also called a *linear system*.

Step 1: Make a table of values.

Step 2: Find an x-value that gives the same y-value for both equations.

x	0	1	2	3
$y = x - 5$	-5	-4	-3	-2
$y = -3x + 7$	7	4	1	-2

∴ The solution is $(3, -2)$.

EXAMPLE 2 Solving a System of Linear Equations Using a Graph

Solve the system. $y = 2x + 3$ Equation 1
$y = -x + 6$ Equation 2

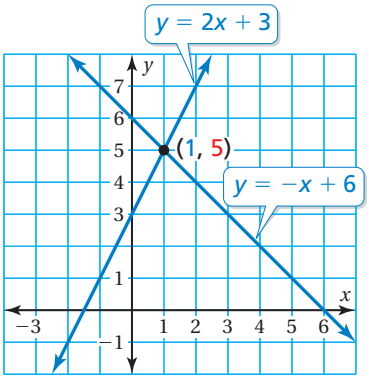
Step 1: Graph each equation.

Step 2: Find the point of intersection. The graphs appear to intersect at $(1, 5)$.

Step 3: Check your solution.

Equation 1
$y = 2x + 3$
$5 \stackrel{?}{=} 2(1) + 3$
$5 = 5$ ✓

Equation 2
$y = -x + 6$
$5 \stackrel{?}{=} -1 + 6$
$5 = 5$ ✓



∴ The solution is $(1, 5)$.

On Your Own

Now You're Ready
Exercises 5–7 and 10–12

Solve the system of linear equations using a table and using a graph.

1. $y = x - 1$
   $y = -x + 3$
2. $y = -5x + 14$
   $y = x - 10$
3. $y = x$
   $y = 2x + 1$

English Spanish

Key Idea

Solving a System of Linear Equations Algebraically

Step 1 Solve both equations for one of the variables.

Step 2 Set the expressions equal to each other and solve for the variable.

Step 3 Substitute back into one of the original equations and solve for the other variable.

EXAMPLE 3 Solving a System of Linear Equations Algebraically

A middle school yearbook committee has 35 members. There are 7 more girls than boys. Use the models to write a system of linear equations. Then solve the system to find the number of boys x and the number of girls y.

$$\text{Number of boys, } x \;+\; \text{Number of girls, } y \;=\; 35$$

$$\text{Number of girls, } y \;=\; \text{Number of boys, } x \;+\; 7$$

The system is $x + y = 35$ and $y = x + 7$.

Step 1: Solve $x + y = 35$ for y.

$y = 35 - x$ Subtract x from each side.

Step 2: Set the expressions equal to each other and solve for x.

$35 - x = x + 7$ Set expressions equal to each other.

$28 = 2x$ Subtract 7 from each side. Add x to each side.

$14 = x$ Divide each side by 2.

Step 3: Substitute $x = 14$ into one of the original equations and solve for y.

$y = x + 7$ Write one of the original equations.

$= 14 + 7$ Substitute 14 for x.

$= 21$ Add.

∴ There are 14 boys and 21 girls on the yearbook committee.

Study Tip

Be sure to check your solutions.

On Your Own

Now You're Ready
Exercises 13–15

4. **WHAT IF?** In Example 3, the yearbook committee has 45 members. Use the models to write a system of linear equations. Then solve the system to find the number of boys x and the number of girls y.

English Spanish

2.5 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** Do the equations $4a - 3b = 5$ and $7b + 2a = -8$ form a system of linear equations? Explain.
2. **REASONING** Can a point in Quadrant II be a break-even point for a system? Explain.

Practice and Problem Solving

Use the table to find the break-even point. Check your solution.

3. $C = 15x + 150$
   $R = 45x$

x	0	1	2	3	4	5	6
C							
R							

4. $C = 24x + 80$
   $R = 44x$

x	0	1	2	3	4	5	6
C							
R							

Solve the system of linear equations using a table.

1

5. $y = x + 4$
   $y = 3x - 1$

6. $y = 1.5x - 2$
   $y = -x + 13$

7. $y = \frac{2}{3}x - 3$
   $y = -2x + 5$

8. **ERROR ANALYSIS** Describe and correct the error in solving the system of linear equations.

x	0	1	2	3
$y = -2x - 1$	-1	-3	-5	-7
$y = x - 7$	-7	-6	-5	-4

The solution is $(-5, -5)$.

9. **CARRIAGE RIDES** The cost C (in dollars) for the care and maintenance of a horse and carriage is $C = 15x + 2000$, where x is the number of rides.

   a. Write an equation for the revenue R in terms of the number of rides.
   b. How many rides are needed for the business to break even?

English Spanish

Solve the system of linear equations using a graph.

2 **10.** $y = 2x + 9$
$y = 6 - x$

11. $y = -x - 4$
$y = \frac{3}{5}x + 4$

12. $y = 2x + 5$
$y = \frac{1}{2}x - 1$

Solve the system of linear equations algebraically.

3 **13.** $x + y = 27$
$y = x + 3$

14. $y - x = 17$
$y = 4x + 2$

15. $x - y = 7$
$0.5x + y = 5$

16. **HOMEWORK** You have 42 math and science problems for homework. You have 10 more math problems than science problems. Use the model to write a system of linear equations. How many problems do you have in each subject?

Number of math problems, x + Number of science problems, y = 42

Number of science problems, y = Number of math problems, x − 10

17. **CANOEING** You and your friend are canoeing. Your position on the river y (in miles) is represented by $y = 3.5x + 28$, where x is in hours. Your friend's position is represented by $y = 2x + 37$.

a. How long will it take you to catch up with your friend?

b. How far will you each have traveled when you catch up with your friend?

18. Critical Thinking You buy x bottles of face paint and y brushes at two stores. The amounts you spend are represented by $10x + 7.5y = 42.5$ and $8x + 6y = 34$. How many bottles of face paint and brushes did you buy?

Fair Game Review What you learned in previous grades & lessons

Decide whether the two equations are equivalent. SECTION 1.2 SECTION 1.3

19. $4n + 1 = n - 8$
$3n = -9$

20. $2a + 6 = 12$
$a + 3 = 6$

21. $7v - \frac{3}{2} = 5$
$14v - 3 = 15$

22. **MULTIPLE CHOICE** Which line has the same slope as $y = \frac{1}{2}x - 3$? SECTION 2.3

(A) $y = -2x + 4$ (B) $y = 2x + 3$ (C) $y - 2x = 5$ (D) $2y - x = 7$