English Spanish

5.3 Angles of Polygons

Essential Question How can you find a formula for the sum of the angle measures of any polygon?

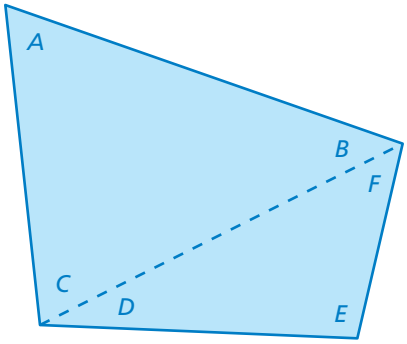
1 ACTIVITY: The Sum of the Angle Measures of a Polygon

Work with a partner. Find the sum of the angle measures of each polygon with *n* sides.

a. **Sample:** Quadrilateral: $n = 4$

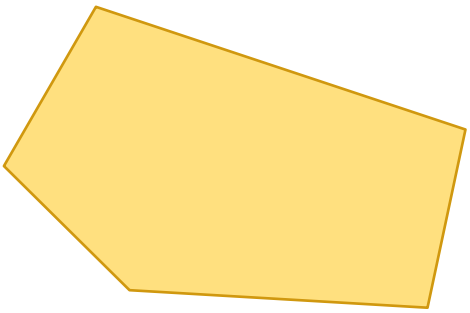
Draw a line that divides the quadrilateral into two triangles.

Because the sum of the angle measures of each triangle is 180°, the sum of the angle measures of the quadrilateral is 360°.

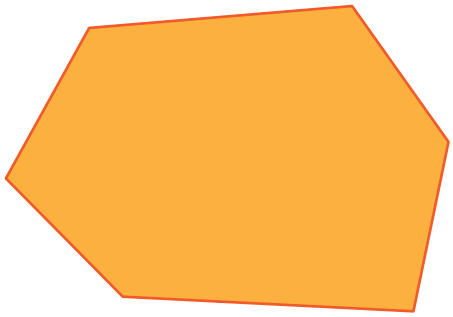


$$(A + B + C) + (D + E + F) = 180° + 180°$$
$$= 360°$$

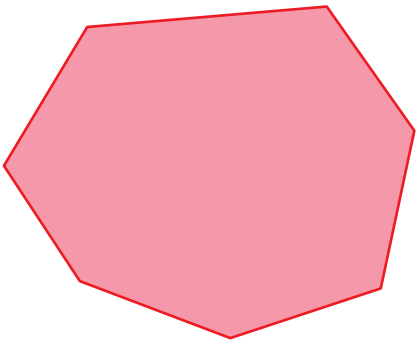
b. Pentagon: $n = 5$



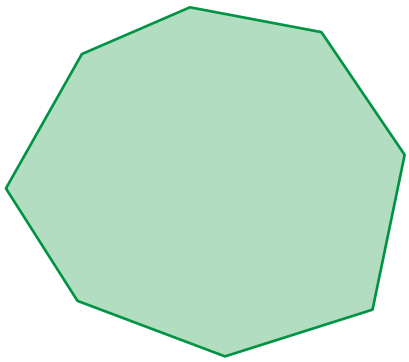
c. Hexagon: $n = 6$



d. Heptagon: $n = 7$



e. Octagon: $n = 8$



2 ACTIVITY: The Sum of the Angle Measures of a Polygon

Work with a partner.

a. Use the table to organize your results from Activity 1.

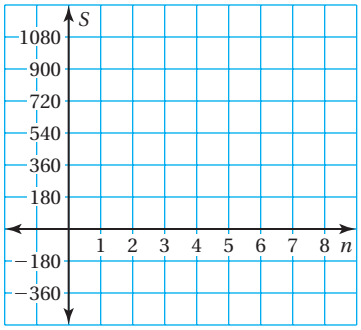
Sides, n	3	4	5	6	7	8
Angle Sum, S						

b. Plot the points in the table in a coordinate plane.

c. Write a linear equation that relates S to n.

d. What is the domain of the function? Explain your reasoning.

e. Use the function to find the sum of the angle measures of a polygon with 10 sides.



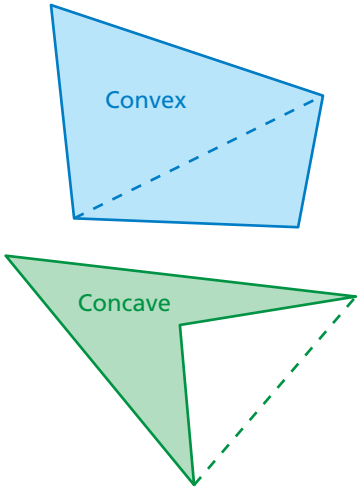
3 ACTIVITY: The Sum of the Angle Measures of a Polygon

Work with a partner.

A polygon is convex if the line segment connecting any two vertices lies entirely inside the polygon. A polygon that is not convex is called concave.

Does the equation you found in Activity 2 apply to concave polygons? Explain.

How can you define the measure of an angle so that your equation applies to *any* polygon?



What Is Your Answer?

4. IN YOUR OWN WORDS How can you find a formula for the sum of the angle measures of any polygon?

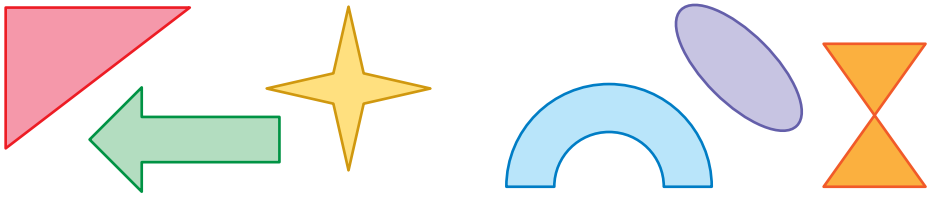
Practice Use what you learned about angles of polygons to complete Exercises 4–6 on page 201.

5.3 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
polygon, *p. 198*
regular polygon, *p. 199*
convex polygon, *p. 200*
concave polygon, *p. 200*

A **polygon** is a closed plane figure made up of three or more line segments that intersect only at their endpoints.



Polygons **Not polygons**

Key Idea

Angle Measures of a Polygon

The sum S of the angle measures of a polygon with n sides is

$$S = (n - 2) \cdot 180°.$$

EXAMPLE 1 Finding the Sum of the Angle Measures of a Polygon

Reading

For polygons whose names you have not learned, you can use the phrase "n-gon," where n is the number of sides. For example, a 15-gon is a polygon with 15 sides.

Find the sum of the angle measures of the school crossing sign.

The sign is in the shape of a pentagon. It has 5 sides.

$S = (n - 2) \cdot 180°$	Write the formula.
$= (5 - 2) \cdot 180°$	Substitute 5 for n.
$= 3 \cdot 180°$	Subtract.
$= 540°$	Multiply.

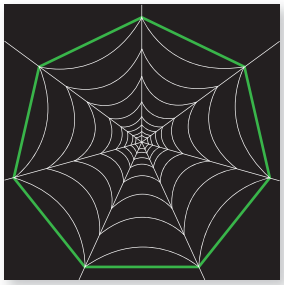
The sum of the angle measures is 540°.

On Your Own

Now You're Ready
Exercises 7–9

Find the sum of the angle measures of the green polygon.

1.



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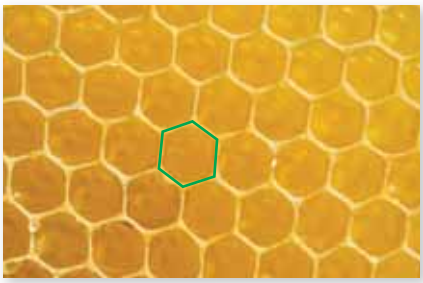

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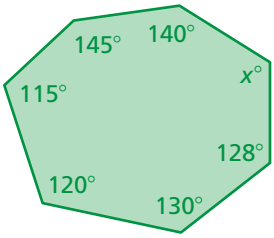
EXAMPLE 2 Finding an Angle Measure of a Polygon



Find the value of x.

Step 1: The polygon has 7 sides. Find the sum of the angle measures.

$S = (n - 2) \cdot 180°$ Write the formula.

$= (7 - 2) \cdot 180°$ Substitute 7 for n.

$= 900°$ Simplify. The sum of the angle measures is 900°.

Step 2: Write and solve an equation.

$$140 + 145 + 115 + 120 + 130 + 128 + x = 900$$
$$778 + x = 900$$
$$x = 122$$

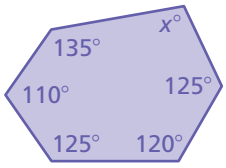
The value of x is 122.

On Your Own

Now You're Ready
Exercises 12–14

Find the value of x.

3.



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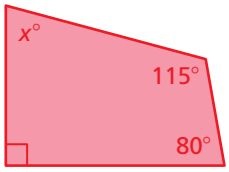

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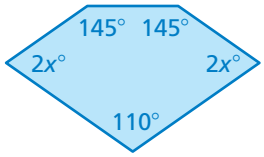
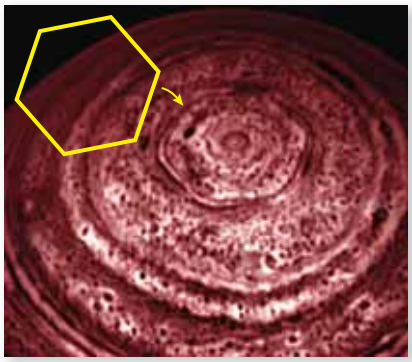


In a **regular polygon**, all of the sides are congruent and all of the angles are congruent.

EXAMPLE 3 Real-Life Application



The hexagon is about 15,000 miles across. Approximately four Earths could fit inside it.

A cloud system discovered on Saturn is in the approximate shape of a regular hexagon. Find the measure of each angle of the hexagon.

Step 1: A hexagon has 6 sides. Find the sum of the angle measures.

$S = (n - 2) \cdot 180°$ Write the formula.

$= (6 - 2) \cdot 180°$ Substitute 6 for n.

$= 720°$ Simplify. The sum of the angle measures is 720°.

Step 2: Divide the sum by the number of angles, 6.

$720° \div 6 = 120°$

The measure of each angle is 120°.

On Your Own

Now You're Ready
Exercises 16–18

Find the measure of each angle of the regular polygon.

6. octagon **7.** decagon **8.** 18-gon

Key Idea

Convex and Concave Polygons

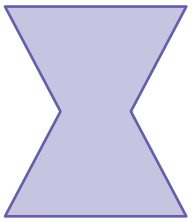
A polygon is **convex** if every line segment connecting any two vertices lies entirely inside the polygon.

A polygon is **concave** if at least one line segment connecting any two vertices lies outside the polygon.

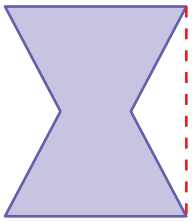
EXAMPLE 4 Identifying Convex and Concave Polygons

Tell whether the polygon is *convex* or *concave*. Explain.

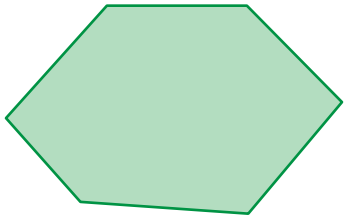
a.



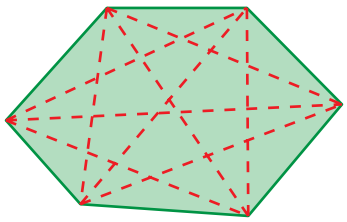
A line segment connecting two vertices lies outside the polygon. So, the polygon is concave.



b.



No line segment connecting two vertices lies outside the polygon. So, the polygon is convex.

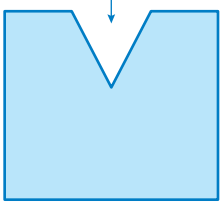


The Meaning of a Word

Concave

To remember the term con**cave**, think of a polygon that is "**cave**d in."

"Caved in"

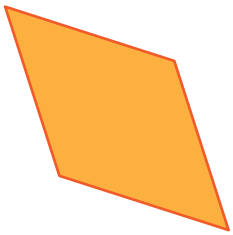


On Your Own

Now You're Ready
Exercises 22–24

Tell whether the polygon is *convex* or *concave*. Explain.

9.



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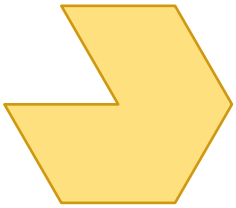

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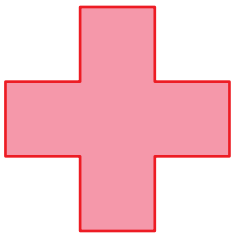

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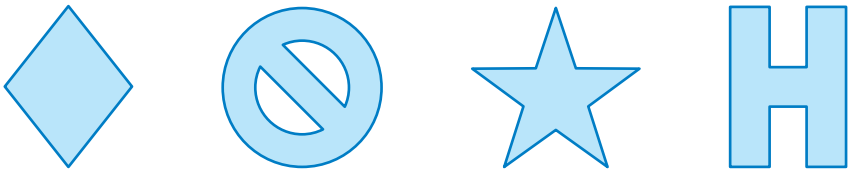
5.3 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** Draw a regular polygon that has three sides.

2. **WHICH ONE DOESN'T BELONG?** Which figure does *not* belong with the other three? Explain your reasoning.

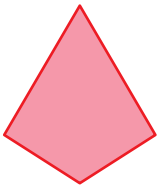


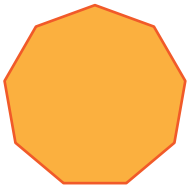
3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What is the measure of an angle of a regular pentagon?	What is the sum of the angle measures of a convex pentagon?
What is the sum of the angle measures of a regular pentagon?	What is the sum of the angle measures of a concave pentagon?

Practice and Problem Solving

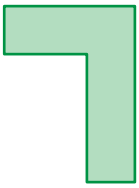
Use triangles to find the sum of the angle measures of the polygon.

4. 

5. 

6. 

Find the sum of the angle measures of the polygon.

1 7. 

8. 

9. 

10. **ERROR ANALYSIS** Describe and correct the error in finding the sum of the angle measures of a 13-gon.

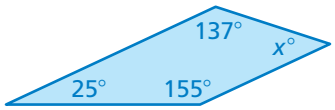
$$S = n \cdot 180°$$
$$= 13 \cdot 180°$$
$$= 2340°$$

11. **NUMBER SENSE** Can a pentagon have angles that measure 120°, 105°, 65°, 150°, and 95°? Explain.

English Spanish

Find the value of x.

2 **12.**



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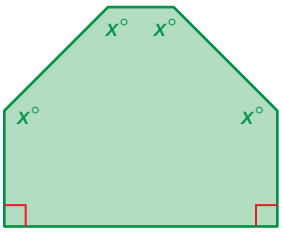

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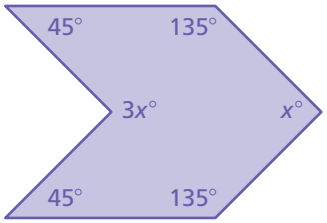
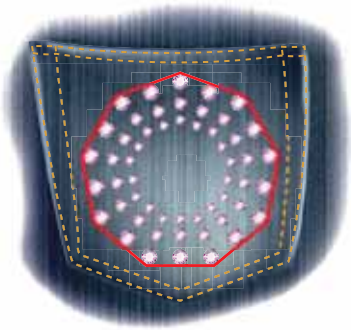


15. **REASONING** The sum of the angle measures in a regular polygon is 1260°. What is the measure of one of the angles of the polygon?

Find the measure of each angle of the regular polygon.

3 **16.**

17.

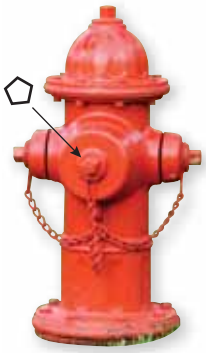


18.

19. **ERROR ANALYSIS** Describe and correct the error in finding the measure of each angle of a regular 20-gon.

$$S = (n - 2) \cdot 180°$$
$$= (20 - 2) \cdot 180°$$
$$= 18 \cdot 180°$$
$$= 3240°$$
$$3240° \div 18 = 180$$

The measure of each angle is 180°.



20. **FIRE HYDRANT** A fire hydrant bolt is in the shape of a regular pentagon.

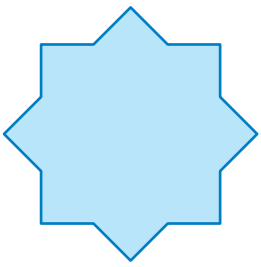
a. What is the measure of each angle?

b. Why are fire hydrants made this way?

21. **PUZZLE** The angles of a regular polygon each measure 165°. How many sides does the polygon have?

Tell whether the polygon is *convex* or *concave*. Explain.

4 **22.**



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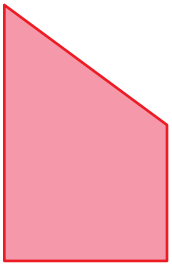


24.

25. **CRITICAL THINKING** Can a concave polygon be regular? Explain.

26. **OPEN-ENDED** Draw a polygon that has congruent sides but is not regular.

27. STAINED GLASS The center of the stained glass window is in the shape of a regular polygon. What is the measure of each angle of the polygon?

28. PENTAGON Draw a pentagon that has two right angles, two 45° angles, and one 270° angle.

29. GAZEBO The floor of a gazebo is in the shape of a heptagon. Four of the angles measure 135°. The other angles have equal measures. Find the measure of each of the remaining angles.

30. MONEY The border of a Susan B. Anthony dollar is in the shape of a regular polygon.

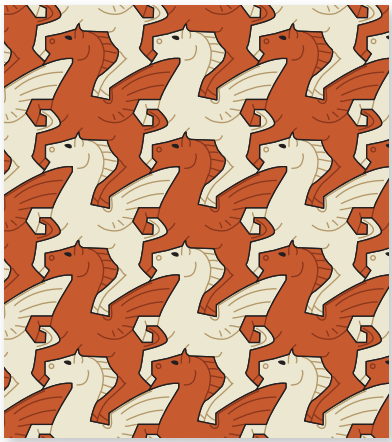
a. How many sides does the polygon have?

b. What is the measure of each angle of the border? Round your answer to the nearest degree.

31. REASONING Copy and complete the table. Does the table represent a linear function? Explain.

Sides of a Regular Polygon, *n*	3	4	5	6	7	8	9	10
Measure of One Angle, *a*								

32. Geometry When tiles can be used to cover a floor with no empty spaces, the collection of tiles is called a *tessellation*.



a. Create a tessellation using equilateral triangles.

b. Find two more regular polygons that form tessellations.

c. Create a tessellation that uses two different regular polygons.

Fair Game Review What you learned in previous grades & lessons

Solve the proportion. SKILLS REVIEW HANDBOOK

33. $\frac{x}{12} = \frac{3}{4}$ **34.** $\frac{14}{21} = \frac{x}{3}$ **35.** $\frac{x}{9} = \frac{2}{6}$ **36.** $\frac{4}{10} = \frac{x}{15}$

37. MULTIPLE CHOICE The ratio of tulips to daisies is 3 : 5. Which of the following could be the total number of tulips and daisies? SKILLS REVIEW HANDBOOK

Ⓐ 6 Ⓑ 10 Ⓒ 15 Ⓓ 16