Solving Inequalities Using Multiplication or Division

Essential Question How can you use multiplication or division to solve an inequality?
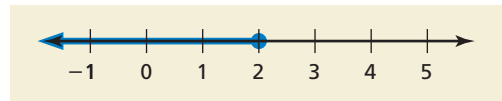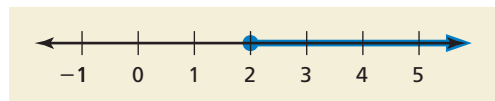
1 ACTIVITY: Using a Table to Solve an Inequality

Work with a partner.

- **Copy and complete the table.**
- **Decide which graph represents the solution of the inequality.**
- **Write the solution of the inequality.**

a. $3x \leq 6$

x	-1	0	1	2	3	4	5
$3x$							
$3x \overset{?}{\leq} 6$							

b. $-2x > 4$

x	-5	-4	-3	-2	-1	0	1
$-2x$							
$-2x \overset{?}{>} 4$							

2 ACTIVITY: Writing a Rule

Work with a partner. Use a table to solve each inequality.

a. $3x > 3$ **b.** $4x \leq 4$ **c.** $-2x \geq 6$ **d.** $-5x < 10$

Write a rule that describes how to solve inequalities like those in Activity 1. Then use your rule to solve each of the four inequalities above.

3 ACTIVITY: Using a Table to Solve an Inequality

Work with a partner.

- **Copy and complete the table.**
- **Decide which graph represents the solution of the inequality.**
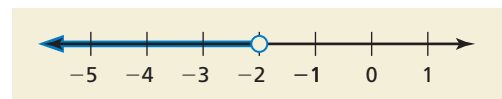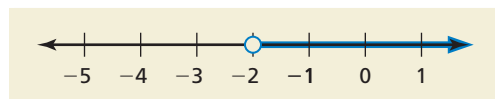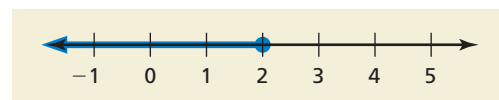- **Write the solution of the inequality.**

a. $\dfrac{x}{2} \geq 1$

x	-1	0	1	2	3	4	5
$\dfrac{x}{2}$							
$\dfrac{x}{2} \overset{?}{\geq} 1$							

b. $\dfrac{x}{-3} < \dfrac{2}{3}$

x	-5	-4	-3	-2	-1	0	1
$\dfrac{x}{-3}$							
$\dfrac{x}{-3} \overset{?}{<} \dfrac{2}{3}$							

4 ACTIVITY: Writing a Rule

Work with a partner. Use a table to solve each inequality.

a. $\dfrac{x}{4} \geq 1$ **b.** $\dfrac{x}{2} < \dfrac{3}{2}$ **c.** $\dfrac{x}{-2} > 2$ **d.** $\dfrac{x}{-5} \leq \dfrac{1}{5}$

Write a rule that describes how to solve inequalities like those in Activity 3. Then use your rule to solve each of the four inequalities above.
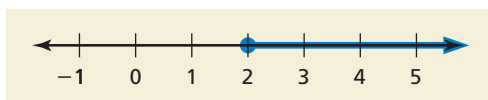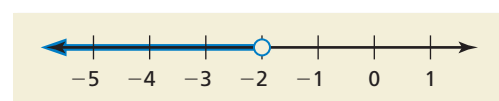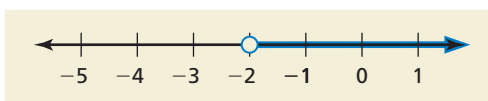
What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you use multiplication or division to solve an inequality?

Practice

Use what you learned about solving inequalities using multiplication or division to complete Exercises 4–9 on page 331.

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BigIdeasMath **✓**com

Key Idea

Remember

Multiplication and division are inverse operations.

Multiplication and Division Properties of Inequality (Case 1)

Words If you multiply or divide each side of an inequality by the same *positive* number, the inequality remains true.

Numbers $-6 < 8$ $6 > -8$

$2 \cdot (-6) < 2 \cdot 8$ $\dfrac{6}{2} > \dfrac{-8}{2}$

$-12 < 16$ $3 > -4$

Algebra $\dfrac{x}{2} < -9$ $4x > -12$

$2 \cdot \dfrac{x}{2} < 2 \cdot (-9)$ $\dfrac{4x}{4} > \dfrac{-12}{4}$

$x < -18$ $x > -3$

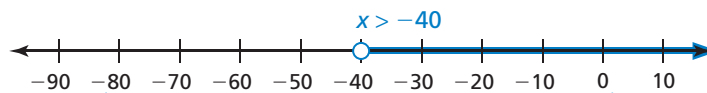These properties are also true for $\leq$ and $\geq$.

EXAMPLE **1** **Solving an Inequality Using Multiplication**

Solve $\dfrac{x}{8} > -5$. Graph the solution.

$\dfrac{x}{8} > -5$ Write the inequality.

Undo the division. ➔ $8 \cdot \dfrac{x}{8} > 8 \cdot (-5)$ Multiply each side by 8.

$x > -40$ Simplify.

∴ The solution is $x > -40$.

$x > -40$

Check: $x = -80$ is *not* a solution.

Check: $x = 0$ is a solution.

On Your Own

Solve the inequality. Graph the solution.

1. $a \div 2 < 4$ **2.** $\dfrac{n}{7} \geq -1$ **3.** $-6.4 \geq \dfrac{w}{5}$

EXAMPLE **2** **Solving an Inequality Using Division**

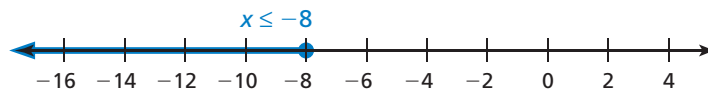**Solve $3x \leq -24$. Graph the solution.**

$$3x \leq -24 \qquad \text{Write the inequality.}$$

| Undo the multiplication. | $\dfrac{3x}{3} \leq \dfrac{-24}{3}$ | Divide each side by 3. |

$$x \leq -8 \qquad \text{Simplify.}$$

The solution is $x \leq -8$.

$x \leq -8$

Check: $x = -10$ is a solution.

Check: $x = 0$ is *not* a solution.

On Your Own

Solve the inequality. Graph the solution.

4. $4b \geq 36$ **5.** $2k > -10$ **6.** $-18 > 1.5q$

Key Idea

Multiplication and Division Properties of Inequality (Case 2)

Words If you multiply or divide each side of an inequality by the same *negative* number, the direction of the inequality symbol must be reversed for the inequality to remain true.

Common Error

A negative sign in an inequality does not necessarily mean you must reverse the inequality symbol.

Only reverse the inequality symbol when you multiply or divide both sides by a negative number.

Numbers

$$-6 < 8 \qquad\qquad 6 > -8$$

$$(-2) \cdot (-6) \; > \; (-2) \cdot 8 \qquad \dfrac{6}{-2} \; < \; \dfrac{-8}{-2}$$

$$12 > -16 \qquad\qquad -3 < 4$$

Algebra

$$\dfrac{x}{-6} < 3 \qquad\qquad -5x > 30$$

$$-6 \cdot \dfrac{x}{-6} \; > \; -6 \cdot 3 \qquad \dfrac{-5x}{-5} \; < \; \dfrac{30}{-5}$$

$$x > -18 \qquad\qquad x < -6$$

These properties are also true for $\leq$ and $\geq$.

EXAMPLE **3** **Solving an Inequality Using Multiplication**

Solve $\dfrac{y}{-3} > 2$. **Graph the solution.**

$$\dfrac{y}{-3} > 2$$ Write the inequality.
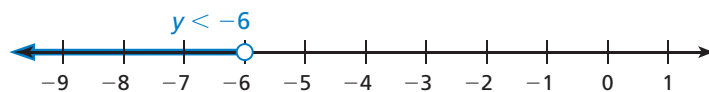
Undo the division. → $-3 \cdot \dfrac{y}{-3} \;<\; -3 \cdot 2$ Multiply each side by -3. Reverse the inequality symbol.

$$y < -6$$ Simplify.

The solution is $y < -6$.

$y < -6$

Check: $y = -9$ is a solution.

Check: $y = 0$ is *not* a solution.

EXAMPLE **4** **Solving an Inequality Using Division**

Solve $-7y \leq -35$. **Graph the solution.**

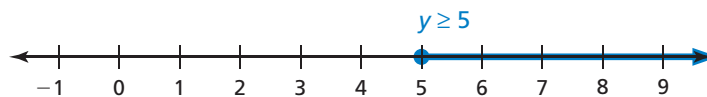$$-7y \leq -35$$ Write the inequality.

Undo the multiplication. → $\dfrac{-7y}{-7} \;\geq\; \dfrac{-35}{-7}$ Divide each side by -7. Reverse the inequality symbol.

$$y \geq 5$$ Simplify.

The solution is $y \geq 5$.

$y \geq 5$

Check: $y = 0$ is *not* a solution.

Check: $y = 6$ is a solution.

● **On Your Own**

Now You're Ready
Exercises 27–35

Solve the inequality. Graph the solution.

7. $\dfrac{p}{-4} < 7$

8. $\dfrac{x}{-5} \leq -5$

9. $1 \geq -\dfrac{1}{10}z$

10. $-9m > 63$

11. $-2r \geq -22$

12. $-0.4y \geq -12$

✓ Vocabulary and Concept Check

1. **VOCABULARY** Explain how to solve $\frac{x}{6} < -5$.

2. **WRITING** Explain how solving $2x < -8$ is different from solving $-2x < 8$.

3. **OPEN-ENDED** Write an inequality that is solved using the Division Property of Inequality where the inequality symbol needs to be reversed.

Practice and Problem Solving

Use a table to solve the inequality.

4. $4x < 4$

5. $-2x \le 2$

6. $-5x > 15$

7. $\frac{x}{-3} \ge 1$

8. $\frac{x}{-2} > \frac{5}{2}$

9. $\frac{x}{4} \le \frac{3}{8}$

Solve the inequality. Graph the solution.

① ② 10. $3n > 18$

11. $\frac{c}{4} \le -9$

12. $1.2m < 12$

13. $-14 > x \div 2$

14. $\frac{w}{5} \ge -2.6$

15. $5 < 2.5k$

16. $4x \le -\frac{3}{2}$

17. $2.6y \le -10.4$

18. $10.2 > \frac{b}{3.4}$

19. **ERROR ANALYSIS** Describe and correct the error in solving the inequality.

✗
$$\frac{x}{2} < -5$$
$$2 \cdot \frac{x}{2} > 2 \cdot (-5)$$
$$x > -10$$

Write the word sentence as an inequality. Then solve the inequality.

20. The quotient of a number and 3 is at most 4.

21. A number divided by 8 is less than -2.

22. Four times a number is at least -12.

23. The product of 5 and a number is greater than 20.

24. **CAMERA** You earn $9.50 per hour at your summer job. Write and solve an inequality that represents the number of hours you need to work in order to buy a digital camera that costs $247.

25. COPIES You have $3.65 to make copies. Write and solve an inequality that represents the number of copies you can make.

26. SPEED LIMIT The maximum speed limit for a school bus is 55 miles per hour. Write and solve an inequality that represents the number of hours it takes to travel 165 miles in a school bus.

Solve the inequality. Graph the solution.

③ ④ 27. $-2n \le 10$

28. $-5w > 30$

29. $\dfrac{h}{-6} \ge 7$

30. $-8 < -\dfrac{1}{3}x$

31. $-2y < -11$

32. $-7d \ge 56$

33. $2.4 > -\dfrac{m}{5}$

34. $\dfrac{k}{-0.5} \le 18$

35. $-2.5 > \dfrac{b}{-1.6}$

36. ERROR ANALYSIS Describe and correct the error in solving the inequality.

✗
$$-4m \ge 16$$
$$\dfrac{-4m}{-4} \ge \dfrac{16}{-4}$$
$$m \ge -4$$

37. CRITICAL THINKING Are all numbers greater than zero solutions of $-x > 0$? Explain.

38. TRUCKING In many states, the maximum height (including freight) of a vehicle is 13.5 feet.

   a. Write and solve an inequality that represents the number of crates that can be stacked vertically on the bed of the truck.
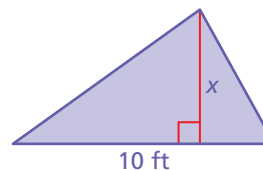   **b.** Five crates are stacked vertically on the bed of the truck. Is this legal? Explain.

28 in.

3.5 ft

Not drawn to scale

Write and solve an inequality that represents the value of x.

39. Area ≥ 102 cm^2

x

12 cm

40. Area < 30 ft^2

x

10 ft

41. TRIP You and three friends are planning a trip. You want to keep the cost below $80 per person. Write and solve an inequality that represents the total cost of the trip.

42. REASONING Explain why the direction of the inequality symbol must be reversed when multiplying or dividing by the same negative number.

43. PROJECT Choose two musical artists to research.

    a. Use the Internet or a magazine to complete the table.

    b. Find the average number of copies sold per month for each CD.

    c. Use the release date to write and solve an inequality that represents the minimum average number of copies sold per month for each CD.

    d. In how many months do you expect the number of copies of the second top selling CD to surpass the current number of copies of the top selling CD?
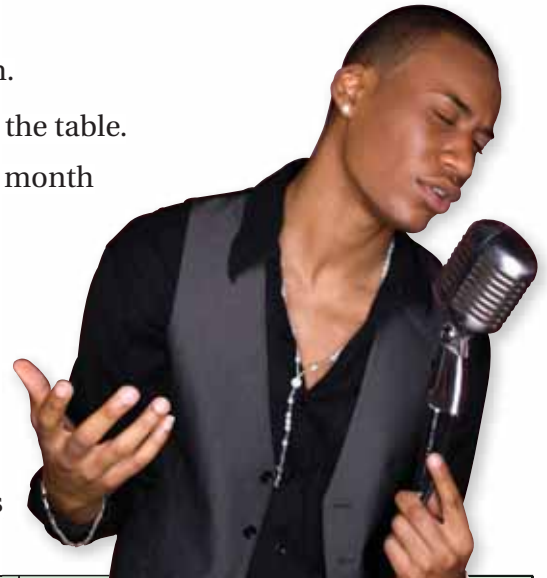
	Artist	Name of CD	Release Date	Current Number of Copies Sold
1.				
2.				

Number Sense Describe all numbers that satisfy *both* inequalities. Include a graph with your description.

44. $3m > -12$ and $2m < 12$

45. $\dfrac{n}{2} \geq -3$ and $\dfrac{n}{-4} \geq 1$

46. $2x \geq -4$ and $2x \geq 4$

47. $\dfrac{m}{-4} > -5$ and $\dfrac{m}{4} < 10$

Fair Game Review *What you learned in previous grades & lessons*

Solve the equation.

48. $-4w + 5 = -11$

49. $4(x - 3) = 21$

50. $\dfrac{v}{6} - 7 = 4$

51. $\dfrac{m + 300}{4} = 96$

52. MULTIPLE CHOICE Which measure can have more than one value for a given data set?

    Ⓐ mean   **Ⓑ** median   **Ⓒ** mode   **Ⓓ** range