

9.4 Zero and Negative Exponents

Essential Question How can you define zero and negative exponents?

1 ACTIVITY: Finding Patterns and Writing Definitions

Work with a partner.

- a. Talk about the following notation.

$$4327 = 4 \cdot 10^3 + 3 \cdot 10^2 + 2 \cdot 10^1 + 7 \cdot 10^{\square}$$

Thousands
Hundreds
Tens
Ones

What patterns do you see in the first three exponents?

Continue the pattern to find the fourth exponent.

How would you define 10^0 ? Explain.

- b. Copy and complete the table.

n	5	4	3	2	1	0
2^n						

What patterns do you see in the first six values of 2^n ?

How would you define 2^0 ? Explain.

- c. Use the Quotient of Powers Property to complete the table.

$\frac{3^5}{3^2} = 3^{5-2} = 3^3 = 27$
$\frac{3^4}{3^2} = 3^{4-2} = \square = \square$
$\frac{3^3}{3^2} = 3^{3-2} = \square = \square$
$\frac{3^2}{3^2} = 3^{2-2} = \square = \square$

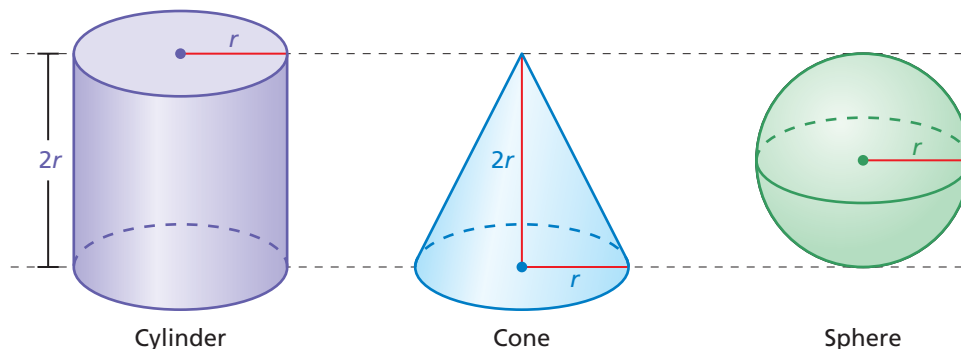
What patterns do you see in the first four rows of the table?

How would you define 3^0 ? Explain.

2 ACTIVITY: Comparing Volumes

Work with a partner.

The quotients show three ratios of the volumes of the solids. Identify each ratio, find its value, and describe what it means.



a. $2\pi r^3 \div \frac{2}{3}\pi r^3 = \square$

b. $\frac{4}{3}\pi r^3 \div \frac{2}{3}\pi r^3 = \square$

c. $2\pi r^3 \div \frac{4}{3}\pi r^3 = \square$

3 ACTIVITY: Writing a Definition

Work with a partner.

Compare the two methods used to simplify $\frac{3^2}{3^5}$. Then describe how you can rewrite a power with a negative exponent as a fraction.

Method 1

$$\frac{3^2}{3^5} = \frac{\overset{1}{3} \cdot \overset{1}{3}}{\underset{1}{3} \cdot \underset{1}{3} \cdot 3 \cdot 3 \cdot 3}$$

$$= \frac{1}{3^3}$$

Method 2

$$\frac{3^2}{3^5} = 3^{2-5}$$

$$= 3^{-3}$$

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you define zero and negative exponents? Give two examples of each.

Practice →

Use what you learned about zero and negative exponents to complete Exercises 5–8 on page 374.



Key Ideas

Zero Exponents

Words Any nonzero number to the zero power is equal to 1. Zero to the zero power, 0^0 , is *undefined*.

Numbers $4^0 = 1$

Algebra $a^0 = 1$, where $a \neq 0$

Negative Exponents

Words For any integer n and any number a not equal to 0, a^{-n} is equal to 1 divided by a^n .

Numbers $4^{-2} = \frac{1}{4^2}$

Algebra $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$

EXAMPLE 1 Evaluating Expressions

a. $3^{-4} = \frac{1}{3^4}$ Definition of negative exponent

$= \frac{1}{81}$ Evaluate power.

b. $(-8.5)^{-4} \cdot (-8.5)^4 = (-8.5)^{-4+4}$ Add the exponents.

$= (-8.5)^0$ Simplify.

$= 1$ Definition of zero exponent

c. $\frac{2^6}{2^8} = 2^{6-8}$ Subtract the exponents.

$= 2^{-2}$ Simplify.

$= \frac{1}{2^2}$ Definition of negative exponent

$= \frac{1}{4}$ Evaluate power.



On Your Own

 **Now You're Ready**
Exercises 9–16

Evaluate the expression.

1. 4^{-2}

2. $(-2)^{-5}$

3. $6^{-8} \cdot 6^8$

4. $\frac{(-3)^5}{(-3)^6}$

5. $\frac{1}{5^7} \cdot \frac{1}{5^{-4}}$

6. $\frac{4^5 \cdot 4^{-3}}{4^2}$

EXAMPLE 2 Simplifying Expressions

a. $-5x^0 = -5(1)$ Definition of zero exponent
 $= -5$ Multiply.

b. $\frac{9y^{-3}}{y^5} = 9y^{-3-5}$ Subtract the exponents.
 $= 9y^{-8}$ Simplify.
 $= \frac{9}{y^8}$ Definition of negative exponent

On Your Own

Now You're Ready
Exercises 20–27

Simplify. Write the expression using only positive exponents.

7. $8x^{-2}$

8. $b^0 \cdot b^{-10}$

9. $\frac{z^6}{15z^9}$

EXAMPLE 3 Real-Life Application

A drop of water leaks from a faucet every second. How many liters of water leak from the faucet in 1 hour?



Drop of water: 50^{-2} L

Convert 1 hour to seconds.

$$1 \cancel{\text{h}} \times \frac{60 \cancel{\text{min}}}{1 \cancel{\text{h}}} \times \frac{60 \text{ sec}}{1 \cancel{\text{min}}} = 3600 \text{ sec}$$

Water leaks from the faucet at a rate of 50^{-2} liter per second. Multiply the time by the rate.

$$\begin{aligned} 3600 \cdot 50^{-2} &= 3600 \cdot \frac{1}{50^2} && \text{Definition of negative exponent} \\ &= 3600 \cdot \frac{1}{2500} && \text{Evaluate power.} \\ &= \frac{3600}{2500} && \text{Multiply.} \\ &= 1\frac{11}{25} = 1.44 && \text{Simplify.} \end{aligned}$$

∴ So, 1.44 liters of water leak from the faucet in 1 hour.

On Your Own

10. **WHAT IF?** In Example 4, the faucet leaks water at a rate of 5^{-5} liter per second. How many liters of water leak from the faucet in 1 hour?

Vocabulary and Concept Check

- VOCABULARY** If a is a nonzero number, does the value of a^0 depend on the value of a ? Explain.
- WRITING** Explain how to evaluate 10^{-3} .
- NUMBER SENSE** Without evaluating, order 5^0 , 5^4 , and 5^{-5} from least to greatest.
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

Rewrite $\frac{1}{3 \cdot 3 \cdot 3}$ using a negative exponent.

Write 3 to the negative third power.

Write $\frac{1}{3}$ cubed as a power.

Write $(-3) \cdot (-3) \cdot (-3)$ as a power.

Practice and Problem Solving

- Use the Quotient of Powers Property to copy and complete the table.
- What patterns do you see?
- How would you define 5^0 ? Why?
- How can you rewrite 5^{-1} as a fraction?

n	4	3	2	1
$\frac{5^n}{5^2}$				

Evaluate the expression.

9. 6^{-2}
10. 158^0
11. $\frac{4^3}{4^5}$
12. $\frac{-3}{(-3)^2}$
13. $(-2)^{-8} \cdot (-2)^8$
14. $3^{-3} \cdot 3^{-2}$
15. $\frac{1}{5^{-3}} \cdot \frac{1}{5^6}$
16. $\frac{(1.5)^2}{(1.5)^{-2} \cdot (1.5)^4}$

- ERROR ANALYSIS** Describe and correct the error in evaluating the expression.

X $(4)^{-3} = (-4)(-4)(-4)$
 $= -64$



- SAND** The mass of a grain of sand is about 10^{-3} gram. About how many grains of sand are in the bag of sand?
- CRITICAL THINKING** How can you write the number 1 as 2 to a power? 10 to a power?

Simplify. Write the expression using only positive exponents.

20. $6y^{-4}$

21. $8^{-2} \cdot a^7$

22. $\frac{9c^3}{c^{-4}}$

23. $\frac{5b^{-2}}{b^{-3}}$

24. $\frac{8x^3}{2x^9}$

25. $3d^{-4} \cdot 4d^4$

26. $m^{-2} \cdot n^3$

27. $\frac{3^{-2} \cdot k^0 \cdot w^0}{w^{-6}}$

METRIC UNITS In Exercises 28–31, use the table.

28. How many millimeters are in a decimeter?

29. How many micrometers are in a centimeter?

30. How many nanometers are in a millimeter?

31. How many micrometers are in a meter?

Unit of Length	Length
decimeter	10^{-1} m
centimeter	10^{-2} m
millimeter	10^{-3} m
micrometer	10^{-6} m
nanometer	10^{-9} m

32. **MICROBES** A species of bacteria is 10 micrometers long. A virus is 10,000 times smaller than the bacteria.

- Using the table above, find the length of the virus in meters.
- Is the answer to part (a) *less than*, *greater than*, or *equal to* one nanometer?

33. **BLOOD DONATION** Every 2 seconds, someone in the United States needs blood. A sample blood donation is shown. ($1 \text{ mm}^3 = 10^{-3} \text{ mL}$)

- One cubic millimeter of blood contains about 10^4 white blood cells. How many white blood cells are in the donation? Write your answer in words.
- One cubic millimeter of blood contains about 5×10^6 red blood cells. How many red blood cells are in the donation? Write your answer in words.
- Compare your answers for parts (a) and (b).

34. **OPEN-ENDED** Write two different powers with negative exponents that have the same value.

35. **Reasoning** The rule for negative exponents states that $a^{-n} = \frac{1}{a^n}$. Explain why this rule does not apply when $a = 0$.



Fair Game Review what you learned in previous grades & lessons

Simplify the expression.

36. $10^3 \cdot 10^6$

37. $10^2 \cdot 10$

38. $\frac{10^8}{10^4}$

39. **MULTIPLE CHOICE** Which data display best shows the variability of a data set?

\checkmark

(A) bar graph

(B) circle graph

(C) scatter plot

(D) box-and-whisker plot