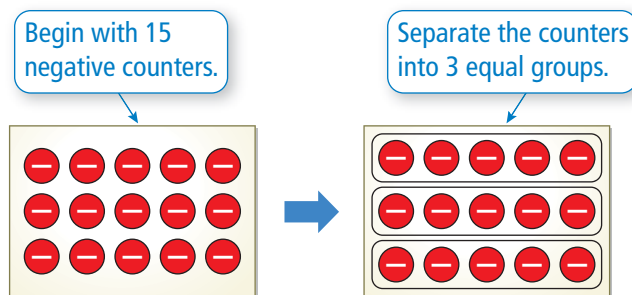


## 1.5 Dividing Integers

**Essential Question** Is the quotient of two integers *positive*, *negative*, or *zero*? How can you tell?

### 1 EXAMPLE: Dividing Integers with Different Signs

Use integer counters to find  $-15 \div 3$ .



Because there are five negative counters in each group,  $-15 \div 3 = -5$ .

### 2 ACTIVITY: Rewriting a Product as a Quotient

Work with a partner. Rewrite the product  $3 \cdot 4 = 12$  as a quotient in two different ways.

*First Way*

12 is equal to 3 groups of .

So,  $12 \div 3 =$  .

*Second Way*

12 is equal to 4 groups of .

So,  $12 \div 4 =$  .

### 3 EXAMPLE: Dividing Integers with Different Signs

Rewrite the product  $-3 \cdot (-4) = 12$  as a quotient in two different ways. What can you conclude?

*First Way*

$$12 \div (-3) = -4$$

*Second Way*

$$12 \div (-4) = -3$$

In each case, when you divide a positive integer by a negative integer, you get a negative integer.

#### 4 EXAMPLE: Dividing Negative Integers

Rewrite the product  $3 \cdot (-4) = -12$  as a quotient in two different ways. What can you conclude?

*First Way*

$$-12 \div (-4) = 3$$

*Second Way*

$$-12 \div (3) = -4$$

⋮ When you divide a negative integer by a negative integer, you get a positive integer. When you divide a negative integer by a positive integer, you get a negative integer.

### Inductive Reasoning

Work with a partner. Complete the table.

	Exercise	Type of Quotient	Quotient	Quotient: Positive, Negative, or Zero
1	5. $-15 \div 3$	Integers with different signs		
2	6. $12 \div 4$			Positive
3	7. $12 \div (-3)$		-4	
4	8. $-12 \div (-4)$	Integers with the same sign		Positive
	9. $-6 \div 2$			
	10. $-21 \div (-7)$			
	11. $10 \div (-2)$			
	12. $12 \div (-6)$			
	13. $0 \div (-15)$			
	14. $0 \div 4$			

### What Is Your Answer?

- IN YOUR OWN WORDS** Is the quotient of two integers *positive*, *negative*, or *zero*? How can you tell?
- Write general rules for dividing (a) two integers with the same sign and (b) two integers with different signs.

#### Practice

Use what you learned about dividing integers to complete Exercises 8–15 on page 32.

## Key Ideas

### Dividing Integers with the Same Sign

**Words** The quotient of two integers with the same sign is positive.

**Numbers**  $8 \div 2 = 4$                        $-8 \div (-2) = 4$

### Dividing Integers with Different Signs

**Words** The quotient of two integers with different signs is negative.

**Numbers**  $8 \div (-2) = -4$                        $-8 \div 2 = -4$

## EXAMPLE 1 Dividing Integers with the Same Sign

Find  $-18 \div (-6)$ .

The integers have the same sign.

$$-18 \div (-6) = 3$$

The quotient is positive.

∴ The quotient is 3.

## EXAMPLE 2 Dividing Integers with Different Signs

Divide.

a.  $75 \div (-25)$

b.  $\frac{-54}{6}$

The integers have different signs.

$$75 \div (-25) = -3$$

$$\frac{-54}{6} = -9$$

The quotient is negative.

∴ The quotient is  $-3$ .

∴ The quotient is  $-9$ .

## On Your Own

Divide.

1.  $14 \div 2$

2.  $-32 \div (-4)$

3.  $-40 \div (-8)$

4.  $0 \div (-6)$

5.  $\frac{-49}{7}$

6.  $\frac{21}{-3}$

Now You're Ready  
Exercises 8–23

### EXAMPLE 3 Evaluating Expressions

#### Remember

Use order of operations when evaluating an expression.



Evaluate  $10 - x^2 \div y$  when  $x = 8$  and  $y = -4$ .

$$\begin{aligned} 10 - x^2 \div y &= 10 - 8^2 \div (-4) && \text{Substitute 8 for } x \text{ and } -4 \text{ for } y. \\ &= 10 - 8 \cdot 8 \div (-4) && \text{Write } 8^2 \text{ as repeated multiplication.} \\ &= 10 - 64 \div (-4) && \text{Multiply 8 and 8.} \\ &= 10 - (-16) && \text{Divide 64 and } -4. \\ &= 26 && \text{Subtract.} \end{aligned}$$

#### On Your Own

Now You're Ready  
Exercises 28–31

Evaluate the expression when  $a = -18$  and  $b = -6$ .

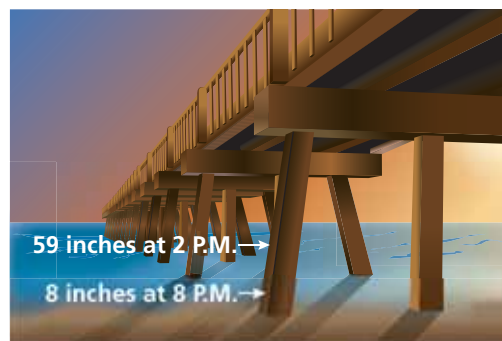
7.  $a \div b$

8.  $\frac{a+6}{3}$

9.  $\frac{b^2}{a} + 4$

### EXAMPLE 4 Real-Life Application

You measure the height of the tide using support beams of a pier. Your measurements are shown in the picture. What is the mean hourly change in the height?



Use a model to solve the problem.

$$\begin{aligned} \text{Mean hourly change} &= \frac{\text{Final height} - \text{Initial height}}{\text{Elapsed Time}} \\ &= \frac{8 - 59}{6} && \text{Substitute. The elapsed time from 2 P.M. to 8 P.M. is 6 hours.} \\ &= \frac{-51}{6} && \text{Subtract.} \\ &= -8.5 && \text{Divide.} \end{aligned}$$

∴ The mean change in the height of the tide is  $-8.5$  inches per hour.

#### On Your Own

10. The height of the tide at the Bay of Fundy in New Brunswick decreases 36 feet in 6 hours. What is the mean hourly change in the height?


**Vocabulary and Concept Check**

- WRITING** What can you tell about two integers when their quotient is positive? negative? zero?
- VOCABULARY** A quotient is undefined. What does this mean?
- OPEN-ENDED** Write two integers whose quotient is negative.
- WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$$\frac{10}{-5}$$

$$\frac{-10}{5}$$

$$\frac{-10}{-5}$$

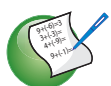
$$-\left(\frac{10}{5}\right)$$

Tell whether the quotient is *positive* or *negative* without dividing.

5.  $-12 \div 4$

6.  $\frac{-6}{-2}$

7.  $15 \div (-3)$



**Practice and Problem Solving**

Divide, if possible.

- |   |   |                      |                     |                      |                      |
|---|---|----------------------|---------------------|----------------------|----------------------|
| 1 | 2 | 8. $4 \div (-2)$     | 9. $21 \div (-7)$   | 10. $-20 \div 4$     | 11. $-18 \div (-6)$  |
|   |   | 12. $\frac{-14}{7}$  | 13. $\frac{0}{6}$   | 14. $\frac{-15}{-5}$ | 15. $\frac{54}{-9}$  |
|   |   | 16. $-33 \div 11$    | 17. $-49 \div (-7)$ | 18. $0 \div (-2)$    | 19. $60 \div (-6)$   |
|   |   | 20. $\frac{-56}{14}$ | 21. $\frac{18}{0}$  | 22. $\frac{65}{-5}$  | 23. $\frac{-84}{-7}$ |


**ERROR ANALYSIS** Describe and correct the error in finding the quotient.

24.



$$\frac{-63}{-9} = -7$$

25.



$$0 \div (-5) = -5$$

- ALLIGATORS** An alligator population in a nature preserve in the Everglades decreases by 60 alligators over 5 years. What is the mean yearly change in the alligator population?
- READING** You read 105 pages of a novel over 7 days. What is the mean number of pages you read each day?

3 **ALGEBRA** Evaluate the expression when  $x = 10$ ,  $y = -2$ , and  $z = -5$ .

28.  $x \div y$

29.  $\frac{10y^2}{z}$

30.  $\left| \frac{xz}{-y} \right|$

31.  $\frac{-x^2 + 6z}{y}$

**Find the mean of the integers.**

32.  $3, -10, -2, 13, 11$

33.  $-26, 39, -10, -16, 12, 31$

**Evaluate the expression.**

34.  $-8 - 14 \div 2 + 5$

35.  $24 \div (-4) + (-2) \cdot (-5)$

36. **PATTERN** Find the next two numbers in the pattern  $-128, 64, -32, 16, \dots$

Explain your reasoning.

37. **HIKING** While hiking along the Croom Tract Loop section of the Florida Trail, a hiker climbs down an 84-foot hill in 4 minutes. What is the mean change in elevation per minute?

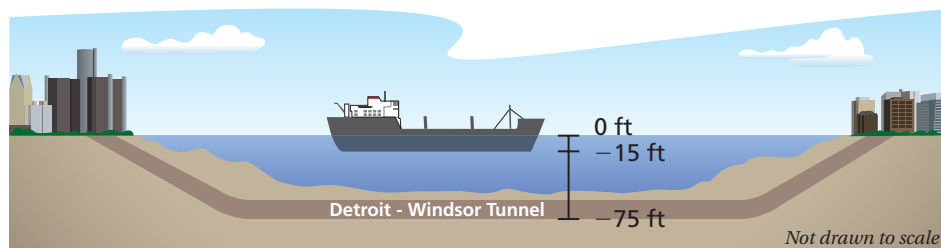
38. **THE MASTERS** In 1997, at the age of 21, Tiger Woods became the youngest golfer to win the Masters Tournament. The table shows his score for each round.

Scorecard	
Round 1	-2
Round 2	-6
Round 3	-7
Round 4	-3

a. Tiger set the tournament record with the lowest total score. What was his total score?

b. What was his mean score per round?

39. **TUNNEL** The Detroit-Windsor Tunnel is an underwater highway that connects the cities of Detroit, Michigan, and Windsor, Ontario. How many times deeper is the roadway than the bottom of the ship?



40. **AMUSEMENT PARK** The regular admission price for an amusement park is \$72. For a group of 15 or more, the admission price is reduced by \$25. How many people need to be in a group to save \$500?

41. **Number Sense** Write five different integers that have a mean of  $-10$ . Explain how you found your answer.



## Fair Game Review What you learned in previous grades & lessons

**Graph the values on a number line. Then order the values from least to greatest.**

42.  $-6, 4, |2|, -1, |-10|$

43.  $3, |0|, |-4|, -3, -8$

44.  $|5|, -2, -5, |-2|, -7$

45. **MULTIPLE CHOICE** What is the value of  $4 \cdot 3 + (12 \div 2)^2$ ?

*O*

(A) 15

(B) 48

(C) 156

(D) 324