

2.5 Systems of Linear Equations

Essential Question How can you solve a system of linear equations?

1 ACTIVITY: Writing a System of Linear Equations

Work with a partner.

Your family starts a bed-and-breakfast in your home. You spend \$500 fixing up a bedroom to rent. Your cost for food and utilities is \$10 per night. Your family charges \$60 per night to rent the bedroom.



- a. Write an equation that represents your costs.

$$\begin{array}{|c|} \hline \text{Cost, } C \\ \hline \text{(in dollars)} \end{array} = \begin{array}{|c|} \hline \$10 \text{ per} \\ \hline \text{night} \end{array} \cdot \begin{array}{|c|} \hline \text{Number of} \\ \hline \text{nights, } x \end{array} + \begin{array}{|c|} \hline \$500 \\ \hline \end{array}$$

- b. Write an equation that represents your revenue (income).

$$\begin{array}{|c|} \hline \text{Revenue, } R \\ \hline \text{(in dollars)} \end{array} = \begin{array}{|c|} \hline \$60 \text{ per} \\ \hline \text{night} \end{array} \cdot \begin{array}{|c|} \hline \text{Number of} \\ \hline \text{nights, } x \end{array}$$

- c. A set of two (or more) linear equations is called a **system of linear equations**. Write the system of linear equations for this problem.

2 ACTIVITY: Using a Table to Solve a System

Use the cost and revenue equations from Activity 1 to find how many nights you need to rent the bedroom before you recover the cost of fixing up the bedroom. This is the *break-even point* for your business.

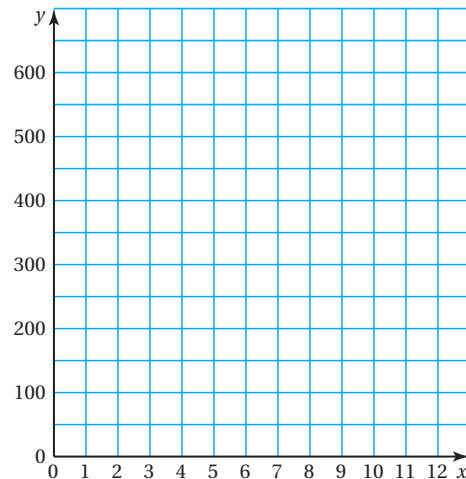
- a. Copy and complete the table.

x	0	1	2	3	4	5	6	7	8	9	10	11
C												
R												

- b. How many nights do you need to rent the bedroom before you break even?

3 ACTIVITY: Using a Graph to Solve a System

- Graph the cost equation from Activity 1.
- In the same coordinate plane, graph the revenue equation from Activity 1.
- Find the point of intersection of the two graphs. The x -value of this point is the number of nights you need to rent the bedroom to break even.



4 ACTIVITY: Using an Equation to Solve a System

- Write the cost equation from Activity 1.

$$C = \boxed{}$$

- Write the revenue equation from Activity 1.

$$R = \boxed{}$$

- The break-even point occurs when $C = R$. Set the expression for C equal to the expression for R . You should obtain an equation with x on both sides. Solve this equation for x . The solution is your break-even point.
- Did you obtain the same break-even point in Activities 2, 3, and 4? If not, check your work. The break-even point should be the same in all three activities.

What Is Your Answer?

- IN YOUR OWN WORDS** How can you solve a system of linear equations?
- When solving a system of linear equations, explain why it is a good idea to use two different ways to find the solution.

Practice

Use what you learned about systems of linear equations to complete Exercises 3 and 4 on page 80.

2.5 Lesson

Key Vocabulary

system of linear equations, p. 78
solution of a system of linear equations, p. 78

A **system of linear equations** is a set of two or more linear equations in the same variables. A **solution of a system of linear equations** in two variables is an ordered pair that makes each equation true.

EXAMPLE 1 Solving a System of Linear Equations Using a Table

Reading

A system of linear equations is also called a *linear system*.

Solve the system. $y = x - 5$ Equation 1

$y = -3x + 7$ Equation 2

Step 1: Make a table of values.

Step 2: Find an x -value that gives the same y -value for both equations.

∴ The solution is $(3, -2)$.

x	0	1	2	3
$y = x - 5$	-5	-4	-3	-2
$y = -3x + 7$	7	4	1	-2

EXAMPLE 2 Solving a System of Linear Equations Using a Graph

Solve the system. $y = 2x + 3$ Equation 1

$y = -x + 6$ Equation 2

Step 1: Graph each equation.

Step 2: Find the point of intersection. The graphs appear to intersect at $(1, 5)$.

Step 3: Check your solution.

Equation 1

$$y = 2x + 3$$

$$5 \stackrel{?}{=} 2(1) + 3$$

$$5 = 5 \quad \checkmark$$

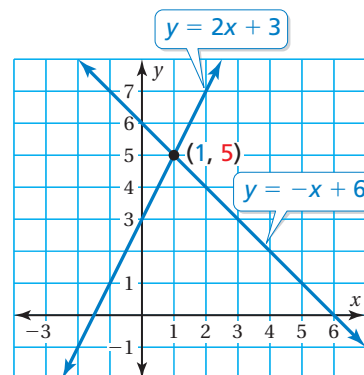
Equation 2

$$y = -x + 6$$

$$5 \stackrel{?}{=} -1 + 6$$

$$5 = 5 \quad \checkmark$$

∴ The solution is $(1, 5)$.



On Your Own

Solve the system of linear equations using a table and using a graph.

1. $y = x - 1$

$y = -x + 3$

2. $y = -5x + 14$

$y = x - 10$

3. $y = x$

$y = 2x + 1$

Now You're Ready

Exercises 5–7
and 10–12

Key Idea

Solving a System of Linear Equations Algebraically

- Step 1** Solve both equations for one of the variables.
- Step 2** Set the expressions equal to each other and solve for the variable.
- Step 3** Substitute back into one of the original equations and solve for the other variable.

EXAMPLE 3 Solving a System of Linear Equations Algebraically



A middle school yearbook committee has 35 members. There are 7 more girls than boys. Use the models to write a system of linear equations. Then solve the system to find the number of boys x and the number of girls y .

Number of boys, x	+	Number of girls, y	=	35		
		Number of girls, y	=	Number of boys, x	+	7

The system is $x + y = 35$ and $y = x + 7$.

Step 1: Solve $x + y = 35$ for y .

$$y = 35 - x \quad \text{Subtract } x \text{ from each side.}$$

Step 2: Set the expressions equal to each other and solve for x .

$$\begin{aligned} 35 - x &= x + 7 && \text{Set expressions equal to each other.} \\ 28 &= 2x && \text{Subtract 7 from each side. Add } x \text{ to each side.} \\ 14 &= x && \text{Divide each side by 2.} \end{aligned}$$

Step 3: Substitute $x = 14$ into one of the original equations and solve for y .

$$\begin{aligned} y &= x + 7 && \text{Write one of the original equations.} \\ &= 14 + 7 && \text{Substitute 14 for } x. \\ &= 21 && \text{Add.} \end{aligned}$$

⋮ There are 14 boys and 21 girls on the yearbook committee.

On Your Own

4. **WHAT IF?** In Example 3, the yearbook committee has 45 members. Use the models to write a system of linear equations. Then solve the system to find the number of boys x and the number of girls y .

Study Tip

Be sure to check your solutions.

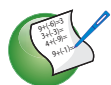
 **Now You're Ready**
Exercises 13–15

2.5 Exercises



Vocabulary and Concept Check

- VOCABULARY** Do the equations $4a - 3b = 5$ and $7b + 2a = -8$ form a system of linear equations? Explain.
- REASONING** Can a point in Quadrant II be a break-even point for a system? Explain.



Practice and Problem Solving

Use the table to find the break-even point. Check your solution.

3. $C = 15x + 150$

$R = 45x$

x	0	1	2	3	4	5	6
C							
R							

4. $C = 24x + 80$

$R = 44x$

x	0	1	2	3	4	5	6
C							
R							

Solve the system of linear equations using a table.

1 5. $y = x + 4$

$y = 3x - 1$

6. $y = 1.5x - 2$

$y = -x + 13$

7. $y = \frac{2}{3}x - 3$

$y = -2x + 5$

8. **ERROR ANALYSIS** Describe and correct the error in solving the system of linear equations.



x	0	1	2	3
$y = -2x - 1$	-1	-3	-5	-7
$y = x - 7$	-7	-6	-5	-4

The solution is $(-5, -5)$.

9. **CARRIAGE RIDES** The cost C (in dollars) for the care and maintenance of a horse and carriage is $C = 15x + 2000$, where x is the number of rides.

- Write an equation for the revenue R in terms of the number of rides.
- How many rides are needed for the business to break even?



Solve the system of linear equations using a graph.

2 10. $y = 2x + 9$

$y = 6 - x$

11. $y = -x - 4$

$y = \frac{3}{5}x + 4$

12. $y = 2x + 5$

$y = \frac{1}{2}x - 1$

Solve the system of linear equations algebraically.

3 13. $x + y = 27$

$y = x + 3$

14. $y - x = 17$

$y = 4x + 2$

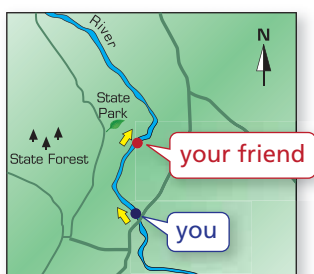
15. $x - y = 7$

$0.5x + y = 5$

16. **HOMEWORK** You have 42 math and science problems for homework. You have 10 more math problems than science problems. Use the model to write a system of linear equations. How many problems do you have in each subject?

Number of math problems, x	+	Number of science problems, y	=	42
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Number of science problems, y	=	Number of math problems, x	-	10
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17. **CANOEING** You and your friend are canoeing. Your position on the river y (in miles) is represented by $y = 3.5x + 28$, where x is in hours. Your friend's position is represented by $y = 2x + 37$.

- How long will it take you to catch up with your friend?
- How far will you each have traveled when you catch up with your friend?

18. **Critical Thinking** You buy x bottles of face paint and y brushes at two stores. The amounts you spend are represented by $10x + 7.5y = 42.5$ and $8x + 6y = 34$. How many bottles of face paint and brushes did you buy?



Fair Game Review What you learned in previous grades & lessons

Decide whether the two equations are equivalent. (Section 1.2 and Section 1.3)

19. $4n + 1 = n - 8$

$3n = -9$

20. $2a + 6 = 12$

$a + 3 = 6$

21. $7v - \frac{3}{2} = 5$

$14v - 3 = 15$

22. **MULTIPLE CHOICE** Which line has the same slope as $y = \frac{1}{2}x - 3$? (Section 2.3)

(A) $y = -2x + 4$

(B) $y = 2x + 3$

(C) $y - 2x = 5$

(D) $2y - x = 7$