English Spanish

2.6 Special Systems of Linear Equations

Essential Question Can a system of linear equations have no solution? Can a system of linear equations have many solutions?

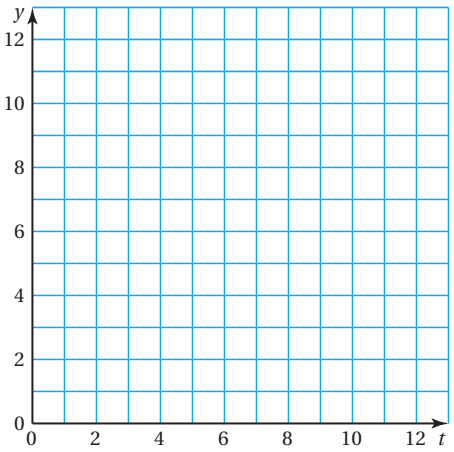
1 ACTIVITY: Writing a System of Linear Equations

Work with a partner.

Your cousin is 3 years older than you. Your ages can be represented by two linear equations.

$y = t$ Your age

$y = t + 3$ Your cousin's age



a. Graph both equations in the same coordinate plane.

b. What is the vertical distance between the two graphs? What does this distance represent?

c. Do the two graphs intersect? If not, what does this mean in terms of your age and your cousin's age?

2 ACTIVITY: Using a Table to Solve a System

Work with a partner. You invest $500 for equipment to make dog backpacks. Each backpack costs you $15 for materials. You sell each backpack for $15.

a. Copy and complete the table for your cost C and your revenue R.

x	0	1	2	3	4	5	6	7	8	9	10
C											
R											

b. When will your company break even? What is wrong?

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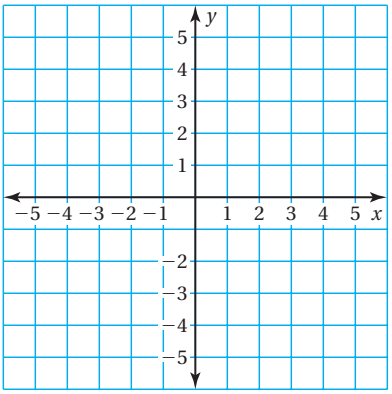
3 ACTIVITY: Using a Graph to Solve a Puzzle

Work with a partner. Let x and y be two numbers. Here are two clues about the values of x and y.

	Words	Equation
Clue 1:	y is 4 more than twice the value of x.	$y = 2x + 4$
Clue 2:	The difference of $3y$ and $6x$ is 12.	$3y - 6x = 12$

a. Graph both equations in the same coordinate plane.



b. Do the two lines intersect? Explain.

c. What is the solution of the puzzle?

d. Use the equation $y = 2x + 4$ to complete the table.

x	0	1	2	3	4	5	6	7	8	9	10
y											

e. Does each solution in the table satisfy *both* clues?

f. What can you conclude? How many solutions does the puzzle have? How can you describe them?

What Is Your Answer?

4. IN YOUR OWN WORDS Can a system of linear equations have no solution? Can a system of linear equations have many solutions? Give examples to support your answers.

Practice Use what you learned about special systems of linear equations to complete Exercises 4 and 5 on page 86.

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2.6 Lesson

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EXAMPLE 1 Solving a Special System of Linear Equations

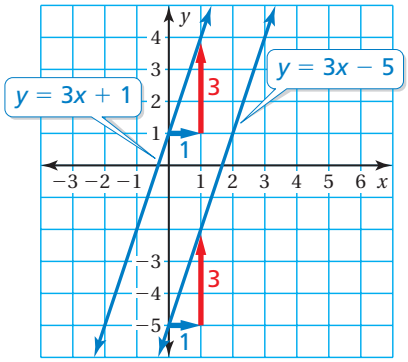
Solve the system.

$y = 3x + 1$ Equation 1
$y = 3x - 5$ Equation 2

Graph each equation.

The lines have the same slope and different y-intercepts. So, the lines are parallel.

Because parallel lines do not intersect, there is no point that is a solution of both equations.



∴ So, the system of linear equations has no solution.

EXAMPLE 2 Solving a Special System of Linear Equations

Solve the system.

$y = -2x + 4$ Equation 1
$4x + 2y = 8$ Equation 2

Write $4x + 2y = 8$ in slope-intercept form.

$4x + 2y = 8$ Write the equation.

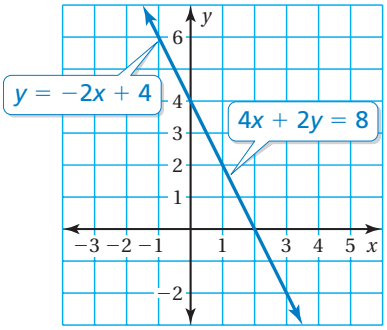
$2y = -4x + 8$ Subtract $4x$ from each side.

$y = -2x + 4$ Divide each side by 2.

The equations are the same.

The solution of the system is all the points on the line $y = -2x + 4$.

∴ So, the system of linear equations has infinitely many solutions.



On Your Own

Now You're Ready
Exercises 6–11

Solve the system of linear equations.

1. $-4x + 4y = 8$
   $y = x + 2$
2. $y = -5x - 2$
   $5x + y = 3$
3. $x + y = 6$
   $y = -x$

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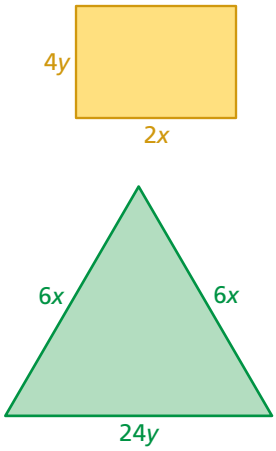
EXAMPLE 3 Solving a Special System of Linear Equations



The perimeter of the rectangle is 36 units. The perimeter of the triangle is 108 units. Write and solve a system of linear equations to find the values of x and y.

Perimeter of rectangle

$2(2x) + 2(4y) = 36$

$4x + 8y = 36$ Equation 1

Perimeter of triangle

$6x + 6x + 24y = 108$

$12x + 24y = 108$ Equation 2

The system of linear equations is $4x + 8y = 36$ and $12x + 24y = 108$.

Write both equations in slope-intercept form.

$4x + 8y = 36$

$8y = -4x + 36$

$y = -\frac{1}{2}x + \frac{9}{2}$

$12x + 24y = 108$

$24y = -12x + 108$

$y = -\frac{1}{2}x + \frac{9}{2}$

The equations are the same. So, the solution of the system is all the points on the line $y = -\frac{1}{2}x + \frac{9}{2}$.

∴ The system of linear equations has infinitely many solutions.

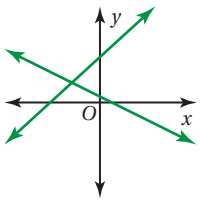
On Your Own

4. **WHAT IF?** What happens to the solution in Example 3 if the perimeter of the rectangle is 54 units? Explain.

Summary

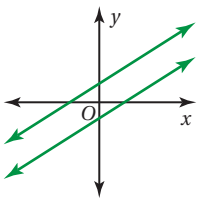
Solutions of Systems of Linear Equations

A system of linear equations can have *one solution*, *no solution*, or *infinitely many solutions*.



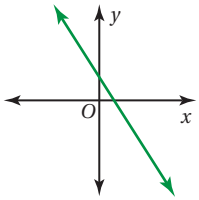
One solution

The lines intersect.



No solution

The lines are parallel.



Infinitely many solutions

The lines are the same.

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2.6 Exercises

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Vocabulary and Concept Check

1. **VOCABULARY** What is the difference between the graph of a system of linear equations that has *no solution* and the graph of a system of linear equations that has *infinitely many solutions*?

2. **NUMBER SENSE** Determine the number of solutions of the system of linear equations without writing the equations in slope-intercept form. Explain your reasoning.

$$2x + y = 5$$
$$4x + 2y = 10$$

3. **REASONING** One equation in a system of linear equations has a slope of -3. The other equation has a slope of 4. How many solutions does the system have? Explain.

Practice and Problem Solving

Let x and y be two numbers. Find the solution of the puzzle.

4. y is $\frac{1}{3}$ more than 4 times the value of x.

The difference of $3y$ and $12x$ is 1.

5. $\frac{1}{2}$ of x plus 3 is equal to y.

x is 6 more than twice the value of y.

Solve the system of linear equations.

1 2 6. $-6x + 3y = 18$
$y = 2x - 2$

7. $y = -\frac{1}{6}x + 5$
$x + 6y = 30$

8. $-x + 2y = -3$
$9x - 3y = -3$

9. $3x + 2y = 0$
$y = x - 5$

10. $y = \frac{4}{9}x + \frac{1}{3}$
$-4x + 9y = 3$

11. $y = -6x + 8$
$12x + 2y = -8$

12. **ERROR ANALYSIS** Describe and correct the error in solving the system of linear equations.

$$y = -2x + 6$$
$$8x + 4y = 24$$

✗
$8x + 4y = 24$
$4y = -8x + 24$
$y = -2x + 6$
The lines have the same slope so there is no solution.

13. **PIG RACE** In a pig race, your pig gets a head start of 3 feet running at a rate of 2 feet per second. Your friend's pig is running at a rate of 2 feet per second. A system of linear equations that represents this situation is $y = 2x + 3$ and $y = 2x$. Will your friend's pig catch up to your pig? Explain.

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Solve the system of linear equations.

14. $y + 4.6x = 5.8x + 0.4$
$-4.8x + 4y = 1.6$

15. $y = \frac{\pi}{3}x + \pi$
$-\pi x + 3y = -6\pi$

16. $-2x + y = 1.3$
$2(0.5x - y) = 4.6$

$4x + 8y = 64$
$8x + 16y = 128$

17. **MONEY** You and a friend both work two different jobs. The system of linear equations represents the total earnings for x hours worked at the first job and y hours worked at the second job. Your friend earns twice as much as you.

a. One week, both of you work 4 hours at the first job. How many hours do you and your friend work at the second job?

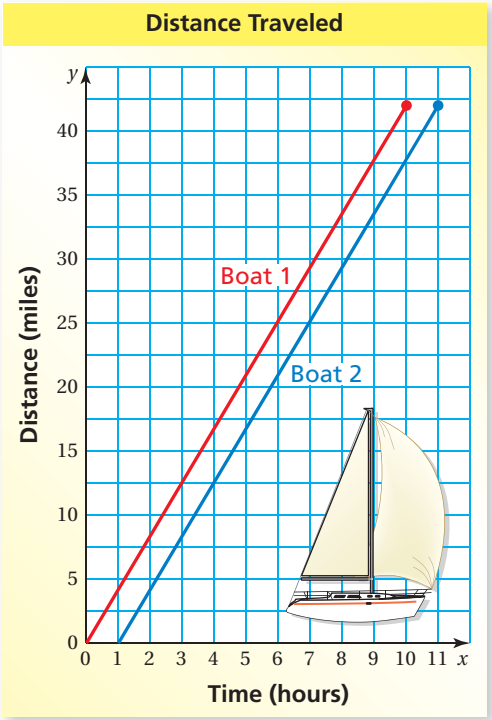
b. Both of you work the same number of hours at the second job. Compare the number of hours you each work at the first job.

18. **BOAT RACE** Two sailboats enter a timed race that is 42 miles long.

a. Which boat left the starting point first?

b. Compare the speeds of the boats.

c. Estimate how long it takes each boat to finish the race.



19. Critical Thinking One equation in a system of linear equations is $y = 3x - 1$.

a. Write a second equation so that $(-2, -7)$ is the only solution of the system.

b. Write a second equation so that the system has no solution.

c. Write a second equation so that the system has infinitely many solutions.

Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution. SECTION 1.3

20. $3x - 5 = 2x + 1$

21. $-3(x - 1) = -8x - 12$

22. $\frac{1}{2}x + 4 = \frac{3}{4}x + 6$

23. **MULTIPLE CHOICE** What is the slope of the line represented by the points in the table? SECTION 2.2

x	0	4	8	12
y	2	5	8	11

Ⓐ $-\frac{3}{4}$ Ⓑ $\frac{3}{4}$ Ⓒ $\frac{4}{3}$ Ⓓ 2