10.2 Product of Powers Property

Essential Question How can you multiply two powers that have the same base?

1 ACTIVITY: Finding Products of Powers

Work with a partner.

a. Copy and complete the table.

Product	Repeated Multiplication Form	Power
$2^2 \cdot 2^4$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	2^6
$(-3)^2 \cdot (-3)^4$	$(-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3)$	$(-3)^6$
$7^3 \cdot 7^2$		
$5.1^1 \cdot 5.1^6$		
$(-4)^2 \cdot (-4)^2$		
$10^3 \cdot 10^5$		
$\left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^5$		

b. **INDUCTIVE REASONING** Describe the pattern in the table. Then write a rule for multiplying two powers that have the same base.
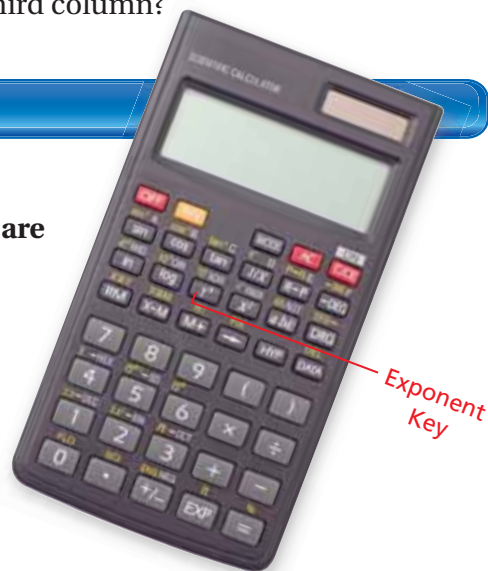
$$a^m \cdot a^n = a^{\boxed{}}$$

c. Use your rule to simplify the products in the first column of the table above. Does your rule give the results in the third column?

2 ACTIVITY: Using a Calculator

Work with a partner.

Some calculators have *exponent keys* that are used to evaluate powers.

**Use a calculator with an exponent key to evaluate the products in Activity 1.**

Exponent Key

ACTIVITY: The Penny Puzzle

Work with a partner.

- The rows y and columns x of a chess board are numbered as shown.
- Each position on the chess board has a stack of pennies. (Only the first row is shown.)
- The number of pennies in each stack is
    $$2^x \cdot 2^y.$$

a. How many pennies are in the stack in location (3, 5)?

b. Which locations have 32 pennies in their stacks?

c. How much money (in dollars) is in the location with the tallest stack?

d. A penny is about 0.06 inch thick. About how tall (in inches) is the tallest stack?
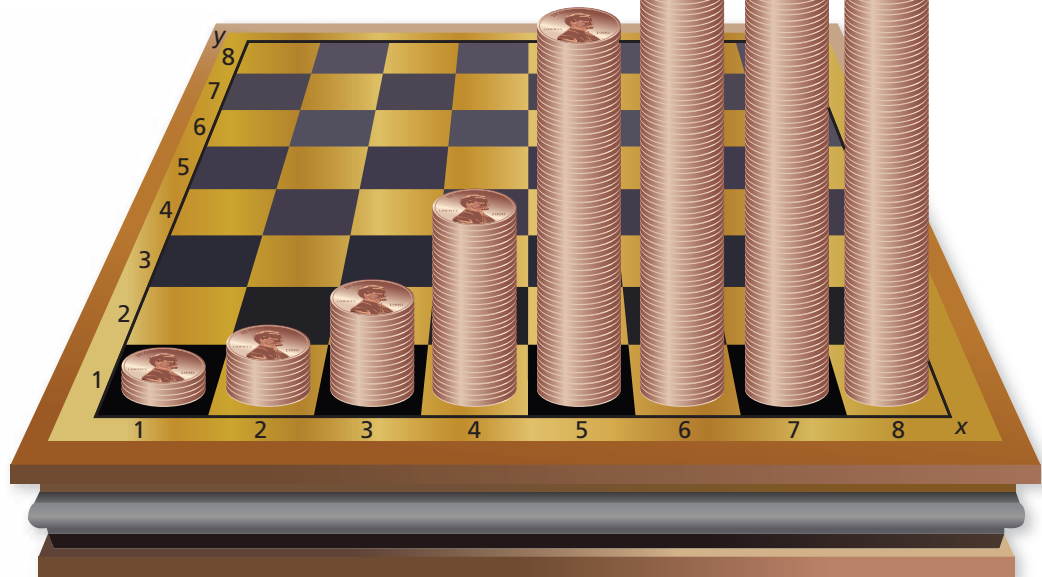
What Is Your Answer?

4. IN YOUR OWN WORDS How can you multiply two powers that have the same base? Give two examples of your rule.

Practice Use what you learned about the Product of Powers Property to complete Exercises 3–5 on page 452.

Key Idea

Product of Powers Property

Words To multiply powers with the same base, add their exponents.

Numbers $4^2 \cdot 4^3 = 4^{2+3} = 4^5$ **Algebra** $a^m \cdot a^n = a^{m+n}$

EXAMPLE 1 Multiplying Powers with the Same Base

a. $2^4 \cdot 2^5 = 2^{4+5}$ The base is 2. Add the exponents.

$= 2^9$ Simplify.

b. $(-5.1)^{-3} \cdot (-5.1)^3 = (-5.1)^{-3+3}$ The base is -5.1. Add the exponents.

$= (-5.1)^0$ Simplify.

$= 1$ Definition of zero exponent

c. $x^3 \cdot x^7 = x^{3+7}$ The base is x. Add the exponents.

$= x^{10}$ Simplify.

EXAMPLE 2 Finding a Power of a Power

a. $(3^4)^3 = 3^4 \cdot 3^4 \cdot 3^4$ Write as repeated multiplication.

$= 3^{4+4+4}$ The base is 3. Add the exponents.

$= 3^{12}$ Simplify.

b. $(w^{-5})^4 = w^{-5} \cdot w^{-5} \cdot w^{-5} \cdot w^{-5}$ Write as repeated multiplication.

$= w^{-5+(-5)+(-5)+(-5)}$ The base is w. Add the exponents.

$= w^{-20}$ Simplify.

$= \dfrac{1}{w^{20}}$ Definition of negative exponent

On Your Own

Now You're Ready
Exercises 3–14

Simplify. Write your answer using only positive exponents.

1. $6^2 \cdot 6^4$

2. $\left(-\dfrac{1}{2}\right)^3 \cdot \left(-\dfrac{1}{2}\right)^6$

3. $z \cdot z^{-12}$

4. $(4^{-4})^3$

5. $(y^2)^4$

6. $((-4)^3)^2$

EXAMPLE **3** **Finding a Power of a Product**

a. $(2x)^3 = 2x \cdot 2x \cdot 2x$ Write as repeated multiplication.

$\quad\quad = (2 \cdot 2 \cdot 2) \cdot (x \cdot x \cdot x)$ Group like bases using properties of multiplication.

$\quad\quad = 2^{1+1+1} \cdot x^{1+1+1}$ The bases are 2 and x. Add the exponents.

$\quad\quad = 2^3 \cdot x^3 = 8x^3$ Simplify.

b. $(xy)^2 = xy \cdot xy$ Write as repeated multiplication.

$\quad\quad = (x \cdot x) \cdot (y \cdot y)$ Group like bases using properties of multiplication.

$\quad\quad = x^{1+1} \cdot y^{1+1}$ The bases are x and y. Add the exponents.

$\quad\quad = x^2y^2$ Simplify.

On Your Own

Now You're Ready
Exercises 17–22

Simplify. Write your answer using only positive exponents.

7. $(-5y)^4$ **8.** $(0.5n)^{-2}$ **9.** $(ab)^5$

EXAMPLE **4** **Standardized Test Practice**

Details ⌃

Local Disk (C:)
Local Disk

Free Space: 16GB

Total Space: 64GB

A gigabyte (GB) of computer storage space is 2^{30} bytes. The details of a computer are shown. How many bytes of total storage space does the computer have?

   (A) 2^{34} **(B)** 2^{36} **(C)** 2^{180} **(D)** 128^{30}

The computer has 64 gigabytes of total storage space. Notice that 64 can be written as a power, 2^6. Use a model to solve the problem.

$$\underset{\text{of bytes}}{\text{Total number}} = \underset{\text{in a gigabyte}}{\text{Number of bytes}} \cdot \underset{\text{gigabytes}}{\text{Number of}}$$

$\quad\quad\quad\quad\quad\quad = 2^{30} \cdot 2^6$ Substitute.

$\quad\quad\quad\quad\quad\quad = 2^{30+6}$ Add exponents.

$\quad\quad\quad\quad\quad\quad = 2^{36}$ Simplify.

⋮∙ The computer has 2^{36} bytes of total storage space. The correct answer is **(B)**.

On Your Own

10. How many bytes of free storage space does the computer have?

Check It Out
Help with Homework
BigIdeasMath com

✓ Vocabulary and Concept Check

1. **REASONING** When should you use the Product of Powers Property?

2. **CRITICAL THINKING** Can you use the Product of Powers Property to multiply powers with different bases? Explain.

Practice and Problem Solving

Simplify. Write your answer using only positive exponents.

①② **3.** $3^2 \cdot 3^2$

4. $8^{10} \cdot 8^{-4}$

5. $(-4)^5 \cdot (-4)^7$

6. $a^{-3} \cdot a^3$

7. $h^6 \cdot h$

8. $\left(\dfrac{2}{3}\right)^{-2} \cdot \left(\dfrac{2}{3}\right)^6$

9. $\left(-\dfrac{5}{7}\right)^8 \cdot \left(-\dfrac{5}{7}\right)^9$

10. $(-2.9) \cdot (-2.9)^7$

11. $\left(5^4\right)^3$

12. $\left(b^{12}\right)^{-3}$

13. $\left(3.8^{-3}\right)^{-4}$

14. $\left(\left(-\dfrac{3}{4}\right)^5\right)^2$

ERROR ANALYSIS Describe and correct the error in simplifying the expression.

15.

✗ $5^2 \cdot 5^9 = (5 \cdot 5)^{2 + 9}$
$= 25^{11}$

16.

✗ $\left(r^6\right)^4 = r^{6 + 4}$
$= r^{10}$

Simplify. Write your answer using only positive exponents.

③ **17.** $(6g)^3$

18. $(2v)^{-5}$

19. $\left(\dfrac{1}{5}k\right)^2$

20. $(1.2m)^4$

21. $(rt)^{-6}$

22. $\left(-\dfrac{3}{4}p\right)^3$

23. CRITICAL THINKING Is $3^2 + 3^3$ equal to 3^5? Explain.

24. ARTIFACT A display case for the artifact is in the shape of a cube. Each side of the display case is three times longer than the width of the artifact.

  a. Write an expression for the volume of the case. Write your answer as a power.

  b. Simplify the expression.

w in.

w in.
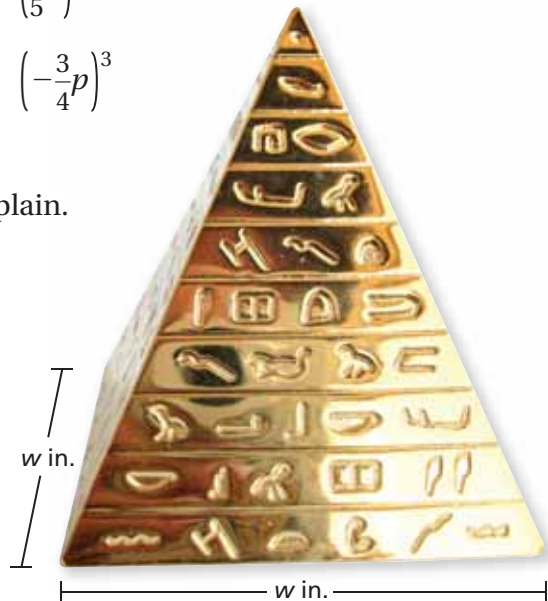
Simplify the expression.

25. $2^4 \cdot 2^5 - (2^2)^2$

26. $16\left(\dfrac{1}{2}x\right)^4$

27. $5^{-2}(5^3 \cdot 5^2)$

28. **CLOUDS** The lowest altitude of an altocumulus cloud is about 3^8 feet. The highest altitude of an altocumulus cloud is about 3 times the lowest altitude. What is the highest altitude of an altocumulus cloud? Write your answer as a power.

29. **PYTHON EGG** The volume V of a python egg is given by the formula $V = \dfrac{4}{3}\pi abc$. For the python egg shown, $a = 2$ inches, $b = 2$ inches, and $c = 3$ inches.

  a. Find the volume of the python egg.

  b. Square the dimensions of the python egg. Then evaluate the formula. How does this volume compare to your answer in part (a)?

30. **PYRAMID** The volume of a square pyramid is $V = \dfrac{1}{3}b^2h$, where b is the length of one side of the base and h is the height of the pyramid. The length of each side of the base increases by 50%. Write a formula for the volume of the new pyramid.

31. **MAIL** The United States Postal Service delivers about $2^6 \cdot 5^3$ pieces of mail each second. There are $2^8 \cdot 3^4 \cdot 5^2$ seconds in 6 days. How many pieces of mail does the United States Postal Service deliver in 6 days? Write your answer as a power.

32. **Critical Thinking** Find the value of x in the equation without evaluating the power.

  a. $2^5 \cdot 2^x = 256$

  b. $\left(\dfrac{1}{3}\right)^2 \cdot \left(\dfrac{1}{3}\right)^x = \dfrac{1}{729}$

Fair Game Review What you learned in previous grades & lessons

Simplify. *(Skills Review Handbook)*

33. $\dfrac{4 \cdot 4}{4}$

34. $\dfrac{5 \cdot 5 \cdot 5}{5}$

35. $\dfrac{2 \cdot 3}{2}$

36. $\dfrac{8 \cdot 6 \cdot 6}{6 \cdot 8}$

37. **MULTIPLE CHOICE** What is the measure of each angle of the regular polygon? *(Section 5.3)*

  A 45°

  B 135°

  C 1080°

  D 1440°