Name ______________________________________ Date __________

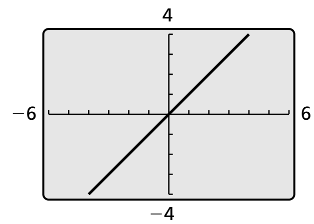
3.6 Transformations of Graphs of Linear Functions

For use with Exploration 3.6

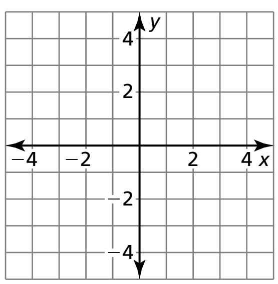
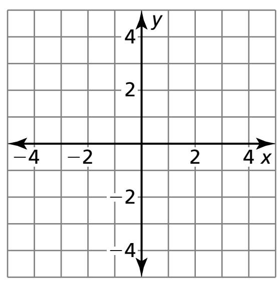
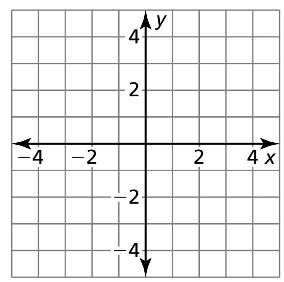
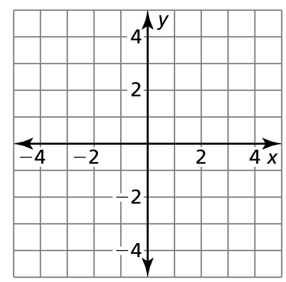
Essential Question How does the graph of the linear function $f(x) = x$ compare to the graphs of $g(x) = f(x) + c$ and $h(x) = f(cx)$?

1 EXPLORATION: Comparing Graphs of Functions

Work with a partner. The graph of $f(x) = x$ is shown. Sketch the graph of each function, along with f, on the same set of coordinate axes. Use a graphing calculator to check your results. What can you conclude?



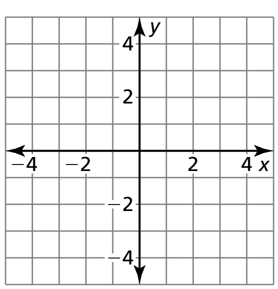
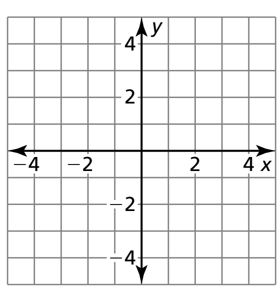
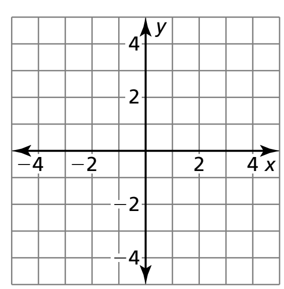
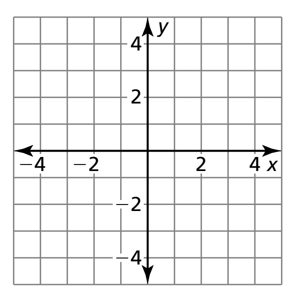
a. $g(x) = x + 4$ **b.** $g(x) = x + 2$ **c.** $g(x) = x - 2$ **d.** $g(x) = x - 4$

2 EXPLORATION: Comparing Graphs of Functions

Work with a partner. Sketch the graph of each function, along with $f(x) = x$, on the same set of coordinate axes. Use a graphing calculator to check your results. What can you conclude?

a. $h(x) = \frac{1}{2}x$ **b.** $h(x) = 2x$ **c.** $h(x) = -\frac{1}{2}x$ **d.** $h(x) = -2x$

Name____ Date__________

3 EXPLORATION: Matching Functions with Their Graphs

Work with a partner. Match each function with its graph. Use a graphing calculator to check your results. Then use the results of Explorations 1 and 2 to compare the graph of k to the graph of $f(x) = x$.

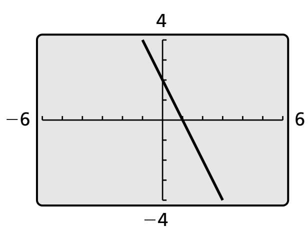
a. $k(x) = 2x - 4$

b. $k(x) = -2x + 2$

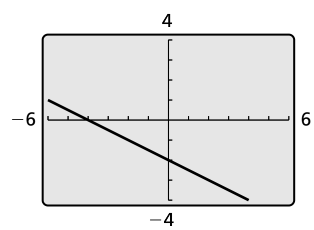
c. $k(x) = \frac{1}{2}x + 4$

d. $k(x) = -\frac{1}{2}x - 2$

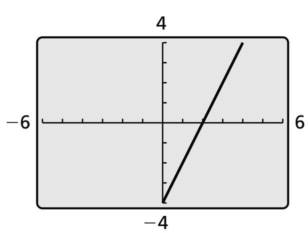
A.



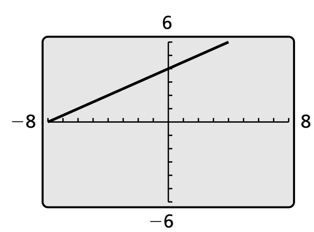
B.



C.



D.



Communicate Your Answer

4. How does the graph of the linear function $f(x) = x$ compare to the graphs of $g(x) = f(x) + c$ and $h(x) = f(cx)$?

Name ____ Date __________

3.6 Practice
For use after Lesson 3.6

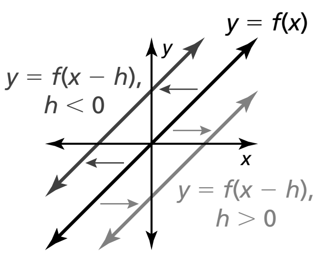
Notes:

Core Concepts

A **translation** is a transformation that shifts a graph horizontally or vertically but does not change the size, shape, or orientation of the graph.

Horizontal Translations

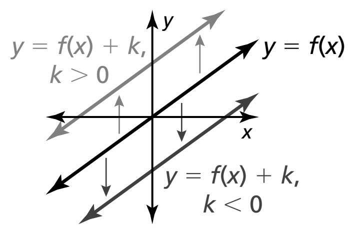
The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$, where $h \neq 0$.



Subtracting h from the *inputs* before evaluating the function shifts the graph left when $h < 0$ and right when $h > 0$.

Vertical Translations

The graph of $y = f(x) + k$ is a vertical translation of the graph of $y = f(x)$, where $k \neq 0$.



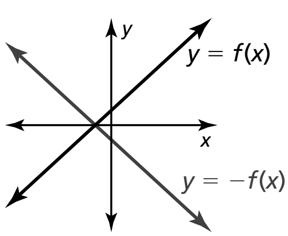
Adding k to the *outputs* shifts the graph down when $k < 0$ and up when $k > 0$.

Notes:

A **reflection** is a transformation that flips a graph over a line called the *line of reflection.*

Reflections in the *x*-axis

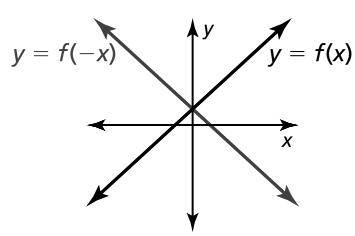
The graph of $y = -f(x)$ is a reflection in the x-axis of the graph of $y = f(x)$.



Multiplying the outputs by -1 changes their signs.

Reflections in the *y*-axis

The graph of $y = f(-x)$ is a reflection in the y-axis of the graph of $y = f(x)$.



Multiplying the inputs by -1 changes their signs.

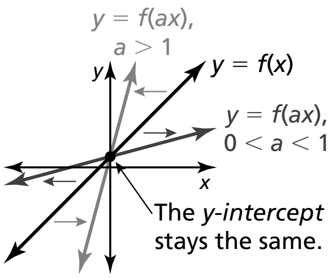
Notes:

Name____ Date

3.6 **Practice (continued)**

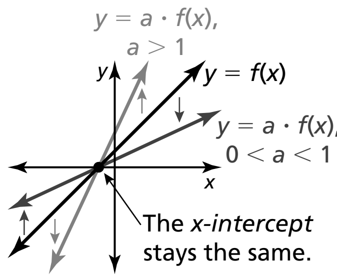
Horizontal Stretches and Shrinks

The graph of $y = f(ax)$ is a horizontal stretch or shrink by a factor of $\frac{1}{a}$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.



Vertical Stretches and Shrinks

The graph of $y = a \bullet f(x)$ is a vertical stretch or shrink by a factor of a of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.



Notes:

Transformations of Graphs

The graph of $y = a \bullet f(x - h) + k$ or the graph of $y = f(ax - h) + k$ can be obtained from the graph of $y = f(x)$ by performing these steps.

Step 1 Translate the graph of $y = f(x)$ horizontally h units.

Step 2 Use a to stretch or shrink the resulting graph from Step 1.

Step 3 Reflect the resulting graph from Step 2 when $a < 0$.

Step 4 Translate the resulting graph from Step 3 vertically k units.

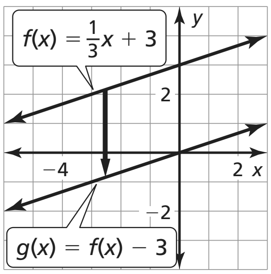
Notes:

Worked-Out Examples

Example #1

Use the graphs of f and g to describe the transformation from the graph of f to the graph of g.

$f(x) = \frac{1}{3}x + 3; g(x) = f(x) - 3$

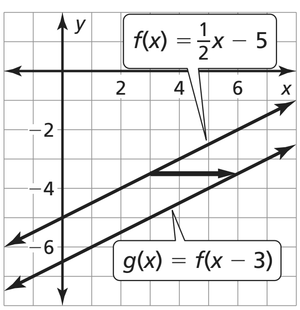


The function g is of the form $y = f(x) + k$, where $k = -3$. So, the graph of g is a vertical translation 3 units down of the graph of f.

Example #2

Use the graphs of f and g to describe the transformation from the graph of f to the graph of g.

$f(x) = \frac{1}{2}x - 5; g(x) = f(x - 3)$



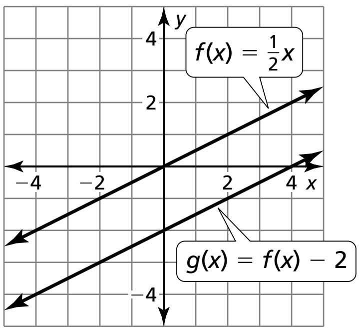
The function g is of the form $y = f(x - h)$, where $h = 3$. So, the graph of g is a horizontal translation 3 units right of the graph of f.

Name____ Date__________

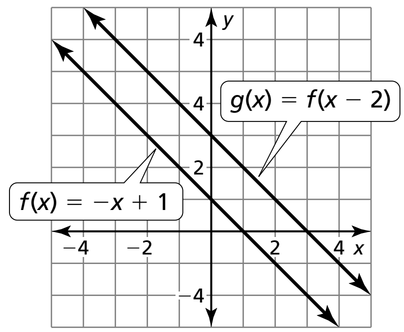
Practice A

In Exercises 1–6, use the graphs of *f* and *g* to describe the transformation from the graph of *f* to the graph of *g*.

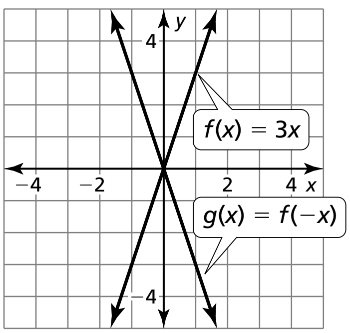
1.



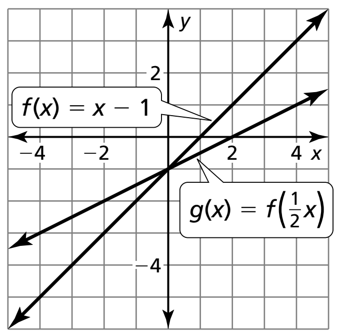
2.



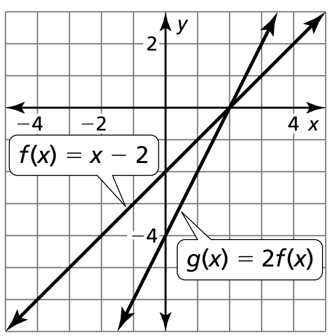
3.



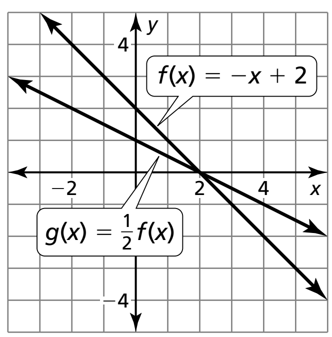
4.



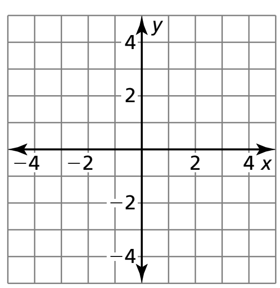
5.



6.



7. Graph $f(x) = x$ and $g(x) = 3x - 2$. Describe the transformations from the graph of f to the graph of g.



Name ____ Date _________

Practice B

In Exercises 1 and 2, use the graphs of *f* and *g* to describe the transformation from the graph of *f* to the graph of *g*.

1. $f(x) = -x - 3;\ g(x) = f(x + 5)$

2. $f(x) = \frac{1}{3}x - 2;\ g(x) = f(x - 6)$

3. The total cost C (in dollars) to rent a 14-foot by 20-foot canopy for d days is given by the function $C(d) = 15d + 30,$ where the setup fee is \$30 and the charge per day is \$15. The setup fee increases by \$20. The new total cost T is given by the function $T(d) = C(d) + 20.$ Describe the transformation from the graph of C to the graph of T.

In Exercises 4 and 5, use the graphs of *f* and *h* to describe the transformation from the graph of *f* to the graph of *h*.

4. $f(x) = -3 - x;\ h(x) = f(-x)$

5. $f(x) = \frac{1}{3}x + 1;\ h(x) = -f(x)$

In Exercises 6 and 7, use the graphs of *f* and *r* to describe the transformation from the graph of *f* to the graph of *r*.

6. $f(x) = 5x - 10;\ r(x) = f\left(\frac{2}{5}x\right)$

7. $f(x) = -\frac{1}{3}x + 2;\ r(x) = 6f(x)$

In Exercises 8–11, use the graphs of *f* and *g* to describe the transformation from the graph of *f* to the graph of *g*.

8. $f(x) = -3x + 5;\ g(x) = f(x - 3)$

9. $f(x) = -2x + 6;\ g(x) = f\left(\frac{4}{3}x\right)$

10. $f(x) = 4x - 3;\ g(x) = \frac{1}{2}f(x)$

11. $f(x) = -2x;\ g(x) = f(x) + 3$

In Exercises 12 and 13, write a function *g* in terms of *f* so that the statement is true.

12. The graph of g is a horizontal shrink by a factor of $\frac{2}{3}$ of the graph of f.

13. The graph of g is a horizontal translation 5 units left of the graph of f.

In Exercises 14–17, graph *f* and *h*. Describe the transformations from the graph of *f* to the graph of *h*.

14. $f(x) = x;\ h(x) = -2x + 1$

15. $f(x) = x;\ h(x) = \frac{3}{2}x + 2$

16. $f(x) = 2x;\ h(x) = 8x - 3$

17. $f(x) = 3x;\ h(x) = -3x - 5$