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5.4 Solving Special Systems of Linear Equations

For use with Exploration 5.4

Essential Question Can a system of linear equations have no solution or infinitely many solutions?

1 EXPLORATION: Using a Table to Solve a System

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. You invest \$450 for equipment to make skateboards. The materials for each skateboard cost \$20. You sell each skateboard for \$20.

a. Write the cost and revenue equations. Then complete the table for your cost *C* and your revenue *R*.

***x* (skateboards)**	0	1	2	3	4	5	6	7	8	9	10
***C* (dollars)**											
***R* (dollars)**											

b. When will your company break even? What is wrong?

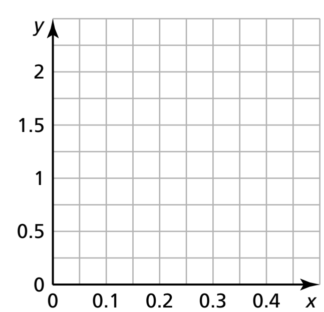
2 EXPLORATION: Writing and Analyzing a System

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. A necklace and matching bracelet have two types of beads. The necklace has 40 small beads and 6 large beads and weighs 10 grams. The bracelet has 20 small beads and 3 large beads and weighs 5 grams. The threads holding the beads have no significant weight.

a. Write a system of linear equations that represents the situation. Let *x* be the weight (in grams) of a small bead and let *y* be the weight (in grams) of a large bead.

b. Graph the system in the coordinate plane shown. What do you notice about the two lines?



c. Can you find the weight of each type of bead? Explain your reasoning.

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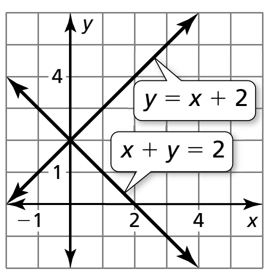
Communicate Your Answer

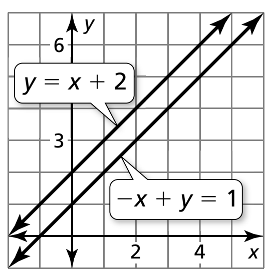
3. Can a system of linear equations have no solution or infinitely many solutions? Give examples to support your answers.

4. Does the system of linear equations represented by each graph have *no solution*, *one solution*, or *infinitely many solutions*? Explain.

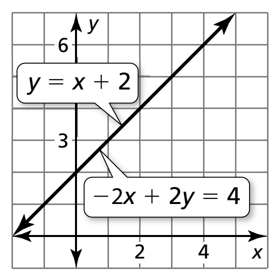
a.



b.



c.



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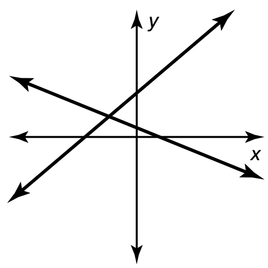
5.4 Practice
For use after Lesson 5.4

Core Concepts

Solutions of Systems of Linear Equations

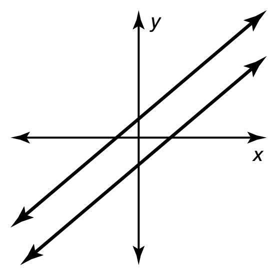
A system of linear equations can have *one solution*, *no solution*, or *infinitely many solutions*.

One solution



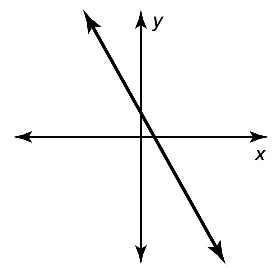
The lines intersect.

No solution



The lines are parallel.

Infinitely many solutions



The lines are the same.

Notes:

Worked-Out Examples

Example #1

Solve the system of linear equations.

$4x + 4y = -8 \qquad -2x - 2y = 4$

Solve by elimination.

Step 1 $4x + 4y = -8$

$-2x - 2y = 4$ **Multiply by 2.**

Step 2

$$\begin{array}{r} 4x + 4y = -8 \\ \underline{-4x - 4y = 8} \\ 0 = 0 \end{array}$$

The equation $0 = 0$ is always true. So, the solutions are all the points on the line $4x + 4y = -8$. The system of linear equations has infinitely many solutions.

Example #2

Solve the system of linear equations.

$9x - 15y = 24 \qquad 6x - 10y = -16$

Solve by elimination.

Step 1

$9x - 15y = 24$ **Multiply by 2.**

$6x - 10y = -16$ **Multiply by −3.**

Step 2

$$\begin{array}{r} 18x - 30y = 48 \\ \underline{-18x + 30y = 48} \\ 0 = 96 \end{array}$$

The equation $0 = 96$ is never true. So, the system of linear equations has no solution.

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5.4 **Practice** **(continued)**

Practice A

In Exercises 1–18, solve the system of linear equations.

1. $y = 3x - 7$
$y = 3x + 4$

2. $y = 5x - 1$
$y = -5x + 5$

3. $2x - 3y = 10$
$-2x + 3y = -10$

4. $x + 3y = 6$
$-x - 3y = 3$

5. $6x + 6y = -3$
$-6x - 6y = 3$

6. $2x - 5y = -3$
$3x + 5y = 8$

7. $2x + 3y = 1$
$-2x + 3y = -7$

8. $4x + 3y = 17$
$-8x - 6y = 34$

9. $3x - 2y = 6$
$-9x + 6y = -18$

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5.4 Practice (continued)

10. $-2x + 5y = -21$
$2x - 5y = 21$

11. $3x - 8y = 3$
$8x - 3y = 8$

12. $18x + 12y = 24$
$3x + 2y = 6$

13. $15x - 6y = 9$
$5x - 2y = 27$

14. $-3x - 5y = 8$
$6x + 10y = -16$

15. $2x - 4y = 2$
$-2x - 4y = 6$

16. $5x + 7y = 7$
$7x + 5y = 5$

17. $y = \frac{2}{3}x + 7$
$y = \frac{2}{3}x - 5$

18. $-3x + 5y = 15$
$9x - 15y = -45$

19. You have \$15 in savings. Your friend has \$25 in savings. You both start saving \$5 per week. Write a system of linear equations that represents this situation. Will you ever have the same amount of savings as your friend? Explain.

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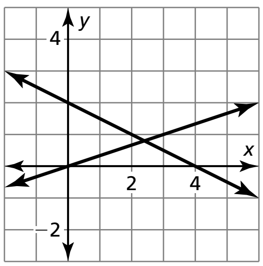
Practice B

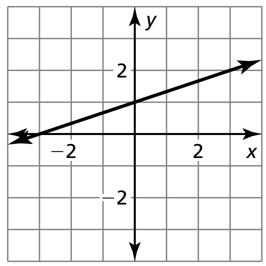
In Exercises 1–3, match the system of linear equations with its graph. Then determine whether the system has *one solution, no solution,* or *infinitely many solutions.*

1. $x - 3y = -3$
$-4x + 12y = 12$

2. $x - 3y = 0$
$x + 2y = 4$

3. $x - 2y = -4$
$3x - 6y = 6$

A.


B.


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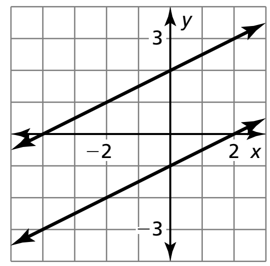

In Exercises 4–9, solve the system of linear equations.

4. $3x - 3y = 6$
$-6x + 6y = -12$

5. $12x - 8y = 10$
$-6x + 4y = 5$

6. $4x - 3y = 16$
$x + y = -3$

7. $6x + 9y = -15$
$4x + 6y = 10$

8. $-x - 4y = 10$
$x + 4y = 10$

9. $-5x + 2y = 3$
$10x - 4y = -6$

In Exercises 10–15, use only the slopes and *y*-intercepts of the graphs of the equations to determine whether the system of linear equations has *one solution, no solution,* or *infinitely many solutions*. Explain.

10. $x - 3y = 9$
$2x - 3y = 9$

11. $-3x + 8y = 32$
$6x - 16y = -64$

12. $2x + 2y = 2$
$9x + 9y = 9$

13. $2x - 4y = -24$
$3x - 6y = -24$

14. $y = -3x + 7$
$3x + 2y = -6$

15. $5x + y = -3$
$2y = -10x - 6$

16. Write a system of three linear equations in two variables so that two of the equations have infinitely many solutions, but the entire system has one solution.

17. Consider the system of linear equations $y = ax + 3$ and $y = \frac{1}{a}x - 2$.

a. If possible, find a value of a so that the system of linear equations has no solution.

b. If possible, find a value of a so that the system of linear equations has one solution.