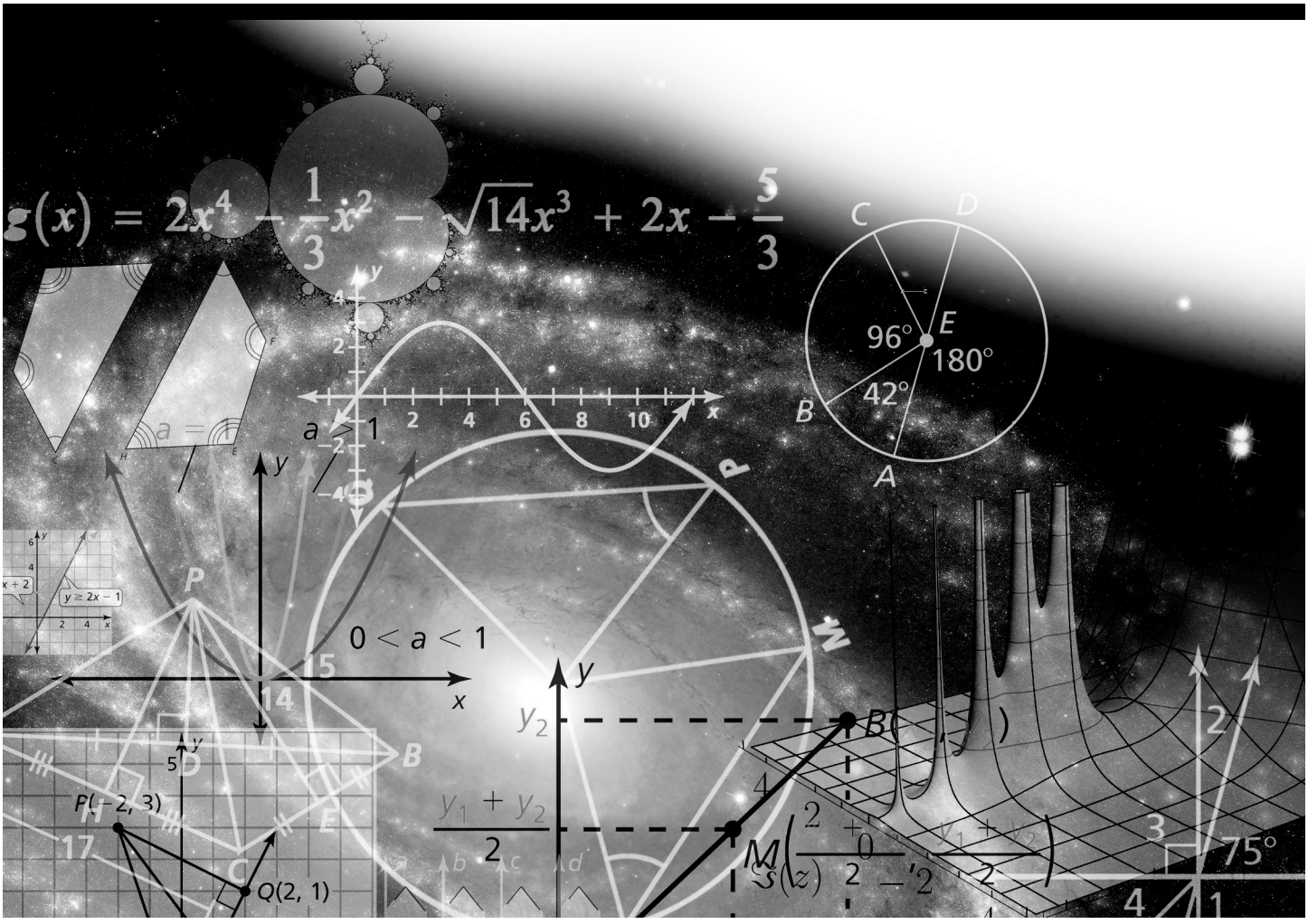
CHAPTER 6

Exponential Functions and Sequences



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Chapter 6 Maintaining Mathematical Proficiency

Evaluate the expression.

1. $(14 + 20 - 6) \div 4 - 6^2$ **2.** $(8 + 4)^2 + (13 - 10 \div 5)$ **3.** $8 \div 4 \bullet 19 + 18 + 13$

4. $3 \bullet 14 \bullet 11 + 4^2 + 19$ **5.** $(21 + 2)(14 - 6) + 3^2$ **6.** $7(3 \bullet 10 - 4^2) + 8$

Evaluate the expression.

7. 64^0 **8.** 4^{-2} **9.** $(-3)^{-3}$ **10.** $7^0 + 5^{-2}$

11. $(-2)^{-6} \bullet 8^0$ **12.** $7^3 \bullet 7^{-3}$ **13.** $10^2 \div (-5)^{-2}$ **14.** $6^{-2} \div 1^9 \bullet 9$

Write an equation for the *n*th term of the arithmetic sequence.

15. $1, 5, 9, 13, \ldots$ **16.** $21, 15, 9, 3, \ldots$ **17.** $-2, 1, 4, 7, \ldots$

18. $8, 6, 4, 2, \ldots$ **19.** $-10, -4, 2, 8, \ldots$ **20.** $16, 8, 0, -8, \ldots$

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6.1 Properties of Exponents

For use with Exploration 6.1

Essential Question How can you write general rules involving properties of exponents?

1 EXPLORATION: Writing Rules for Properties of Exponents

Work with a partner.

a. What happens when you multiply two powers with the same base? Write the product of the two powers as a single power. Then write a *general rule* for finding the product of two powers with the same base.

i. $(2^2)(2^3) =$ _____

ii. $(4^1)(4^5) =$ _____

iii. $(5^3)(5^5) =$ _____

iv. $(x^2)(x^6) =$ _____

b. What happens when you divide two powers with the same base? Write the quotient of the two powers as a single power. Then write a *general rule* for finding the quotient of two powers with the same base.

i. $\frac{4^3}{4^2} =$ _____

ii. $\frac{2^5}{2^2} =$ _____

iii. $\frac{x^6}{x^3} =$ _____

iv. $\frac{3^4}{3^4} =$ _____

c. What happens when you find a power of a power? Write the expression as a single power. Then write a *general rule* for finding a power of a power.

i. $(2^2)^4 =$ _____

ii. $(7^3)^2 =$ _____

iii. $(y^3)^3 =$ _____

iv. $(x^4)^2 =$ _____

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6.1 Properties of Exponents (continued)

1 EXPLORATION: Writing Rules for Properties of Exponents (continued)

d. What happens when you find a power of a product? Write the expression as the product of two powers. Then write a *general rule* for finding a power of a product.

i. $(2 \bullet 5)^2 =$ _____ **ii.** $(5 \bullet 4)^3 =$ _____

iii. $(6a)^2 =$ _____ **iv.** $(3x)^2 =$ _____

e. What happens when you find a power of a quotient? Write the expression as the quotient of two powers. Then write a *general rule* for finding a power of a quotient.

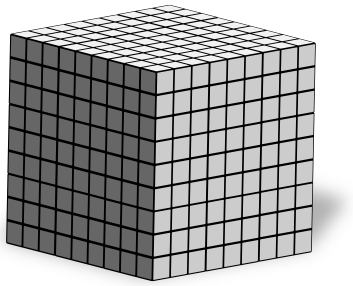
i. $\left(\frac{2}{3}\right)^2 =$ _____ **ii.** $\left(\frac{4}{3}\right)^3 =$ _____

iii. $\left(\frac{x}{2}\right)^3 =$ _____ **iv.** $\left(\frac{a}{b}\right)^4 =$ _____

Communicate Your Answer

2. How can you write general rules involving properties of exponents?

3. There are 3^3 small cubes in the cube below. Write an expression for the number of small cubes in the large cube at the right.



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6.1 Practice
For use after Lesson 6.1

Core Concepts

Zero Exponent

Words For any nonzero number a, $a^0 = 1$. The power 0^0 is undefined.

Numbers $4^0 = 1$ **Algebra** $a^0 = 1$, where $a \neq 0$

Negative Exponents

Words For any integer n and any nonzero number a, a^{-n} is the reciprocal of a^n.

Numbers $4^{-2} = \frac{1}{4^2}$ **Algebra** $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$

Notes:

Product of Powers Property

Let a be a real number, and let m and n be integers.

Words To multiply powers with the same base, add their exponents.

Numbers $4^6 \bullet 4^3 = 4^{6+3} = 4^9$ **Algebra** $a^m \bullet a^n = a^{m+n}$

Quotient of Powers Property

Let a be a nonzero real number, and let m and n be integers.

Words To divide powers with the same base, subtract their exponents.

Numbers $\frac{4^6}{4^3} = 4^{6-3} = 4^3$ **Algebra** $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Power of a Power Property

Let a be a real number, and let m and n be integers.

Words To find a power of a power, multiply the exponents.

Numbers $\left(4^6\right)^3 = 4^{6\bullet 3} = 4^{18}$ **Algebra** $\left(a^m\right)^n = a^{mn}$

Notes:

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6.1 Practice (continued)

Power of a Product Property

Let a and b be real numbers, and let m be an integer.

Words To find a power of a product, find the power of each factor and multiply.

Numbers $(3 \bullet 2)^5 = 3^5 \bullet 2^5$ **Algebra** $(ab)^m = a^m b^m$

Power of a Quotient Property

Let a and b be real numbers with $b \neq 0$, and let m be an integer.

Words To find the power of a quotient, find the power of the numerator and the power of the denominator and divide.

Numbers $\left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5}$ **Algebra** $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$

Notes:

Worked-Out Examples

Example #1

Evaluate the expression.

$\frac{2^{-4}}{4^0}$

$\frac{2^{-4}}{4^0} = \frac{1}{1 \bullet 2^4} = \frac{1}{16}$

Example #2

Simplify the expression. Write your answer using only positive exponents.

$\frac{13x^{-5}y^0}{5^{-3}z^{-10}}$

$\frac{13x^{-5}y^0}{5^{-3}z^{-10}} = \frac{13 \bullet 1 \bullet 5^3 \bullet z^{10}}{x^5} = \frac{13 \bullet 125 \bullet z^{10}}{x^5} = \frac{1625z^{10}}{x^5}$

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6.1 Practice (continued)

Practice A

In Exercises 1–8, evaluate the expression.

1. 3^0 **2.** $(-2)^0$ **3.** 3^{-4} **4.** $(-4)^{-3}$

5. $\frac{2^{-3}}{5^0}$ **6.** $\frac{-3^{-2}}{2^{-3}}$ **7.** $\frac{4^{-1}}{-7^0}$ **8.** $\frac{3^{-1}}{(-5)^0}$

In Exercises 9–23, simplify the expression. Write your answer using only positive exponents.

9. z^0 **10.** a^{-8} **11.** $6a^0b^{-2}$

12. $14m^{-4}n^0$ **13.** $\frac{3^{-2}r^{-3}}{s^0}$ **14.** $\frac{2^3a^{-3}}{8^{-1}b^{-5}c^0}$

15. $\frac{3^5}{3^3}$ **16.** $\frac{(-2)^7}{(-2)^5}$ **17.** $(-5)^3 \bullet (-5)^3$

18. $\left(q^5\right)^3$ **19.** $\left(a^{-4}\right)^2$ **20.** $\frac{c^4 \bullet c^3}{c^6}$

21. $(-4d)^4$ **22.** $(-3f)^{-3}$ **23.** $\left(\frac{4}{x}\right)^{-3}$

24. A rectangular prism has length x, width $\frac{x}{2}$, and height $\frac{x}{3}$. Which of the expressions represent the volume of the prism? Select all that apply.

A. $6^{-1}x^3$ **B.** $6^{-1}x^{-3}$ **C.** $\left(6x^{-3}\right)^{-1}$ **D.** $2^{-1} \bullet 3^{-1} \bullet x^3$

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Practice B

In Exercises 1–6, evaluate the expression.

1. 5^{-4}

2. $(-5)^{-4}$

3. $\dfrac{7^{-1}}{-8^{0}}$

4. $\dfrac{8^{-1}}{(-4)^{0}}$

5. $\dfrac{-2^{-4}}{3^{-3}}$

6. $\dfrac{6^{-2}}{(-1)^{-4}}$

In Exercises 7–21, simplify the expression. Write your answer using only positive exponents.

7. $\dfrac{7^{-2}m^{0}}{n^{-4}}$

8. $\dfrac{(-9)^{0}\,j^{-1}k^{-4}}{2^{0}}$

9. $\dfrac{5^{-2}w^{0}}{y^{-10}}$

10. $\dfrac{t^{-5}}{8^{-2}s^{-3}}$

11. $\dfrac{3^{-2}a^{-1}}{9^{-1}b^{-2}c^{0}}$

12. $\dfrac{17x^{0}y^{-8}}{4^{-2}z^{-6}}$

13. $\left(p^{6}\right)^{3}$

14. $\left(q^{-4}\right)^{5}$

15. $5^{3} \bullet 5^{-7}$

16. $-4 \bullet (-4)^{-2}$

17. $\dfrac{x^{7}}{x^{4}} \bullet x^{2}$

18. $\dfrac{v^{5} \bullet v^{3}}{v^{2}}$

19. $\left(-8t^{2}\right)^{3}$

20. $\left(-\dfrac{q^{4}}{5}\right)^{-3}$

21. $\left(\dfrac{1}{3h^{5}}\right)^{-4}$

In Exercises 22 and 23, simplify the expression. Write your answer using only positive exponents.

22. $\left(\dfrac{5x^{-4}y^{3}}{2x^{2}y^{0}}\right)^{2} \bullet \left(\dfrac{4xy}{y^{3}}\right)^{2}$

23. $\left(\dfrac{2a^{0}b^{-4}}{b^{3}}\right)^{4} \bullet \left(\dfrac{a^{3}b^{-2}}{3b^{4}a^{-4}}\right)^{3}$

In Exercises 24 and 25, evaluate the expression. Write your answer in scientific notation and standard form.

24. $\left(4.3 \times 10^{-4}\right)\left(6 \times 10^{7}\right)$

25. $\dfrac{1.2 \times 10^{-3}}{4.8 \times 10^{-10}}$

26. Find x and y when $b^{x}b^{y} = b^{8}$ and $b^{4x}b^{-2y} = b^{2}$. Explain how you found your answer.

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