

6.3**Exponential Growth and Decay**

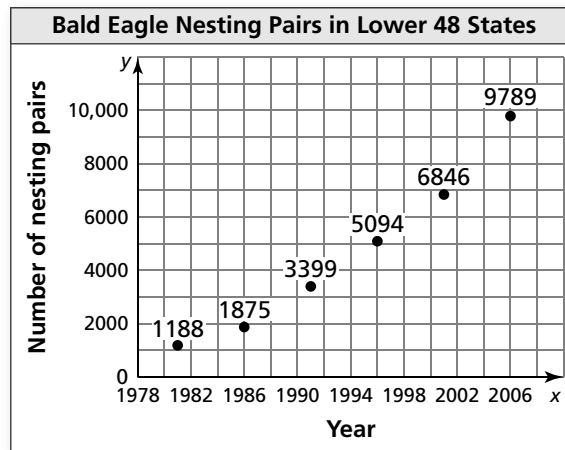
For use with Exploration 6.3

Essential Question What are some of the characteristics of exponential growth and exponential decay functions?

1 EXPLORATION: Predicting a Future Event

Work with a partner. It is estimated, that in 1782, there were about 100,000 nesting pairs of bald eagles in the United States. By the 1960s, this number had dropped to about 500 nesting pairs. In 1967, the bald eagle was declared an endangered species in the United States. With protection, the nesting pair population began to increase. Finally, in 2007, the bald eagle was removed from the list of endangered and threatened species.

Describe the pattern shown in the graph. Is it exponential growth? Assume the pattern continues. When will the population return to that of the late 1700s? Explain your reasoning.



6.3 Exponential Growth and Decay (continued)**2 EXPLORATION:** Describing a Decay Pattern

Work with a partner. A forensic pathologist was called to estimate the time of death of a person. At midnight, the body temperature was 80.5°F and the room temperature was a constant 60°F . One hour later, the body temperature was 78.5°F .

- a. By what percent did the difference between the body temperature and the room temperature drop during the hour?
- b. Assume that the original body temperature was 98.6°F . Use the percent decrease found in part (a) to make a table showing the decreases in body temperature. Use the table to estimate the time of death.

Time (h)								
Temperature difference ($^{\circ}\text{F}$)								
Body temperature ($^{\circ}\text{F}$)								

Communicate Your Answer

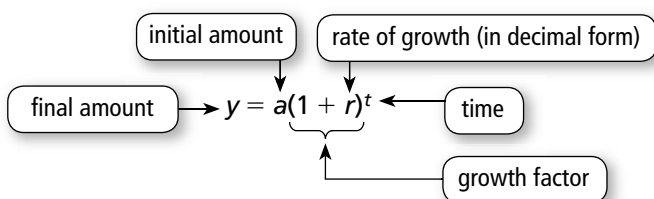
3. What are some of the characteristics of exponential growth and exponential decay functions?
4. Use the Internet or some other reference to find an example of each type of function. Your examples should be different than those given in Explorations 1 and 2.
 - a. exponential growth
 - b. exponential decay

6.3**Practice**

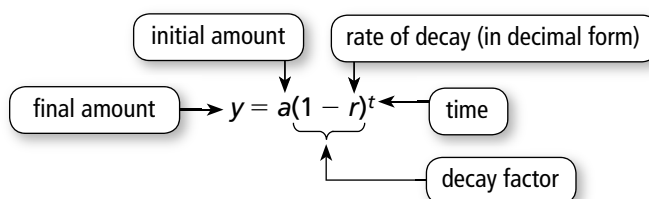
For use after Lesson 6.3

Core Concepts**Exponential Growth Functions**

A function of the form $y = a(1 + r)^t$, where $a > 0$ and $r > 0$, is an **exponential growth function**.

**Notes:****Exponential Decay Functions**

A function of the form $y = a(1 - r)^t$, where $a > 0$ and $0 < r < 1$, is an **exponential decay function**.

**Notes:**

6.3 Practice (continued)**Compound Interest**

Compound interest is the interest earned on the principal *and* on previously earned interest. The balance y of an account earning compound interest is

$$y = P\left(1 + \frac{r}{n}\right)^{nt}$$

P = principal (initial amount)

r = annual interest rate (in decimal form)

t = time (in years)

n = number of times interest is compounded per year

Notes:**Worked-Out Examples****Example #1**

Identify the initial amount a and the rate of growth r (as a percent) of the exponential function. Evaluate the function when $t = 5$. Round your answer to the nearest tenth.

$$f(t) = 1500(1.074)^t$$

$$1 + r = 1.074$$

$$\begin{array}{r} -1 \\ \hline \end{array} \qquad \begin{array}{r} -1 \\ \hline \end{array}$$

$$r = 0.074$$

The initial amount is $a = 1500$, and the rate of growth is $r = 0.074$, or 7.4%.

$$f(t) = 1500(1.074)^t$$

$$f(5) = 1500(1.074)^5$$

$$\approx 1500(1.429)$$

$$\approx 2143.4$$

So, the value of $f(t)$ is about 2143.4 when $t = 5$.

Example #2

Write a function that represents the situation.

The initial amount is 35,000, and the rate of growth is 4%, or 0.04.

$$y = a(1 + r)^t$$

$$y = 35,000(1 + 0.04)^t$$

$$y = 35,000(1.04)^t$$

Your salary can be represented by $y = 35,000(1.04)^t$.

6.3 Practice (continued)**Practice A**

1. In 2005, there were 100 rabbits in Polygon Park. The population increased by 11% each year.
 - a. Write an exponential growth function that represents the population t years after 2005.
 - b. What will the population be in 2025? Round your answer to the nearest whole number.

In Exercises 2–5, determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain.

2.

x	y
0	20
1	30
2	45
3	67.5

3.

x	y
-1	160
0	40
1	10
2	2.5

4.

x	y
1	32
2	22
3	12
4	2

5.

x	y
-1	4
0	10
1	25
2	62.5

In Exercises 6–8, determine whether each function represents *exponential growth* or *exponential decay*. Identify the percent rate of change.

6. $y = 4(0.95)^t$

7. $y = 500(1.08)^t$

8. $w(t) = \left(\frac{3}{4}\right)^t$

In Exercises 9 and 10, write a function that represents the balance after t years.

9. \$3000 deposit that earns 6% annual interest compounded quarterly.
10. \$5000 deposit that earns 7.2% annual interest compounded monthly.

Practice B

In Exercises 1–3, identify the initial amount a and the rate of growth r (as a percent) of the exponential function. Evaluate the function when $t = 5$. Round your answer to the nearest tenth.

1. $f(t) = 220(1.015)^t$ 2. $p(t) = 5.5(1.5)^t$ 3. $h(t) = 2.5^t$

In Exercises 4 and 5, write a function that represents the situation.

4. A college's tuition of \$135 per credit hour increases by 5% each year.
5. A bee population of 3000 increases by 40% every year.

In Exercises 6–8, identify the initial amount a and the rate of decay r (as a percent) of the exponential function. Evaluate the function when $t = 3$. Round your answer to the nearest tenth.

6. $f(t) = 1420(0.895)^t$ 7. $y = \left(\frac{3}{5}\right)^t$ 8. $y = 9.2\left(\frac{1}{3}\right)^t$

In Exercises 9 and 10, write a function that represents the situation.

9. A \$25,000 car decreases by 16.7% each year.
10. A company's annual revenue of \$487,000 decreases by 4.2% each year.

In Exercises 11 and 12, determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain.

11.

x	2	4	6	8
y	5	10	15	20

12.

x	1	5	9	13
y	81	54	36	24

13. The table shows the total numbers of shares of an initial public offering purchased t days after it opens on the stock market.

x	1	2	3	4
y	6250	2500	1000	400

- a. Determine whether the table represents an exponential growth function, an exponential decay function, or neither.
b. How many shares were purchased after the stock had been opened for 6 days?