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6.5 Modeling with Exponential and Logarithmic Functions

For use with Exploration 6.5

Essential Question How can you recognize polynomial, exponential, and logarithmic models?

1 EXPLORATION: Recognizing Different Types of Models

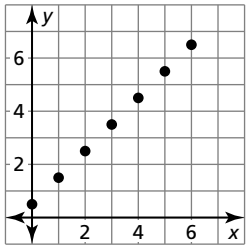
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Match each type of model with the appropriate scatter plot. Use a regression program to find a model that fits the scatter plot.

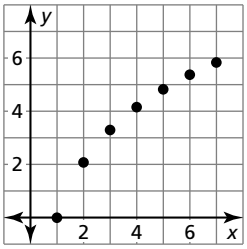
a. linear (positive slope) **b.** linear (negative slope) **c.** quadratic

d. cubic **e.** exponential **f.** logarithmic

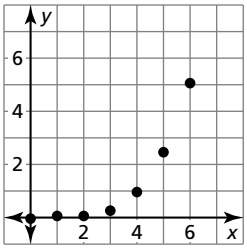
A.



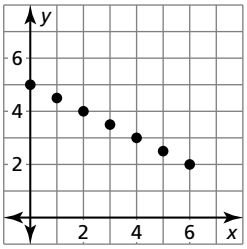
B.



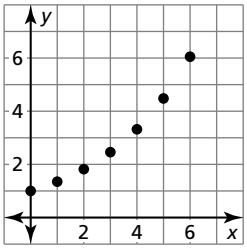
C.



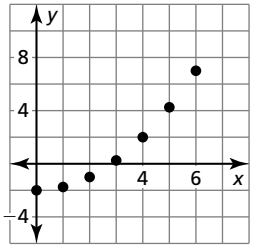
D.



E.



F.



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2 EXPLORATION: Exploring Gaussian and Logistic Models

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Two common types of functions that are related to exponential functions are given. Use a graphing calculator to graph each function. Then determine the domain, range, intercept, and asymptote(s) of the function.

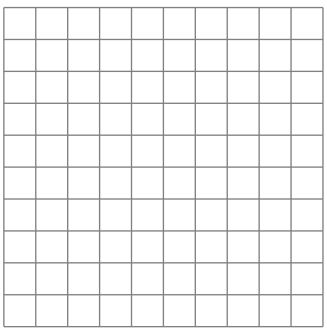
a. Gaussian Function: $f(x) = e^{-x^2}$

b. Logistic Function: $f(x) = \frac{1}{1 + e^{-x}}$

Communicate Your Answer

3. How can you recognize polynomial, exponential, and logarithmic models?

4. Use the Internet or some other reference to find real-life data that can be modeled using one of the types given in Exploration 1. Create a table and a scatter plot of the data. Then use a regression program to find a model that fits the data.



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6.5 Practice

For use after Lesson 6.5

Notes:

Worked-Out Examples

Example #1

Write an exponential function $y = ab^x$ whose graph passes through the given points.

(3, 27), (5, 243)

Step 1 Substitute the coordinates (3, 27) and (5, 243) into $y = ab^x$.

$$27 = ab^3$$
$$243 = ab^5$$

Step 2 Solve for a in Equation 1 to obtain $a = \frac{27}{b^3}$ and substitute this expression for a into Equation 2.

$$243 = \left(\frac{27}{b^3}\right)b^5$$
$$243 = 27b^2$$
$$9 = b^2$$
$$3 = b$$

Step 3 Determine that $a = \frac{27}{b^3} = \frac{27}{3^3} = 1$.

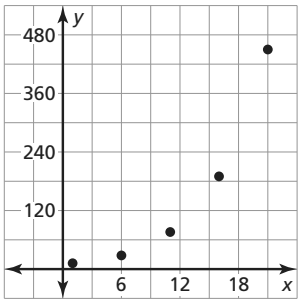
So, the exponential function is $y = 1 \cdot (3^x) = 3^x$.

Example #2

Determine whether the data show an exponential relationship. Then write a function that models the data.

x	1	6	11	16	21
y	12	28	76	190	450

Sample answer: Make a scatter plot of the data.



The data appear exponential. Choose any two points to write a model, such as (1, 12) and (6, 28). Substitute the coordinates of these points into $y = ab^x$.

$$12 = ab^1$$
$$28 = ab^6$$

Solve for a in the first equation to obtain $a = \frac{12}{b}$. Substitute to obtain $b \approx 1.19$ and $a \approx 10.13$.

So, an exponential function that models the data is $y = 10.13(1.19^x)$.

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6.5 Practice (continued)

Practice A

In Exercises 1 and 2, determine the type of function represented by the table. Explain your reasoning.

1.

x	6	7	8	9	10	11
y	34	47	62	79	98	119

2.

x	-5	-3	-1	1	3	5
y	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{9}{5}$	$\frac{27}{5}$	$\frac{81}{5}$	$\frac{243}{5}$

In Exercises 3–6, write an exponential function $y = ab^x$ whose graph passes through the given points.

3. $(1, 12), (3, 108)$

4. $(-1, 2), (3, 32)$

5. $(2, 9), (4, 324)$

6. $(-2, 2), (1, 0.25)$

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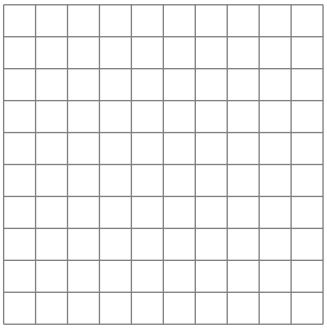
6.5 Practice (continued)

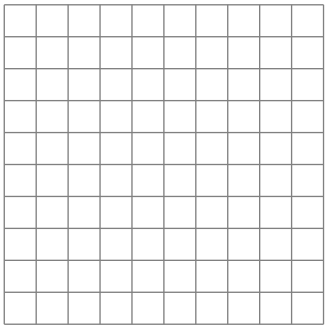
7. An Olympic swimmer starts selling a new type of goggles. The table shows the number y of goggles sold during a 6-month period.

Months, *x*	1	2	3	4	5	6
Goggles sold, *y*	28	47	64	79	97	107

a. Create a scatterplot of the data.



b. Create a scatterplot of the data pairs $(x, \ln y)$ to show that an exponential model should be a good fit for the original data pairs (x, y). Write a function that models the data.



c. Use a graphing calculator to write an exponential model for the data.

d. Use each model to predict the number of goggles sold after 1 year.

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Practice B

In Exercises 1 and 2, determine the type of function represented by the table. Explain your reasoning.

1.

x	0	2	4	6	8
y	$\frac{1}{8}$	$\frac{1}{2}$	2	8	32

2.

x	0	1	2	3
y	8	12	18	27

In Exercises 3–8, write an exponential function $y = ab^x$ whose graph passes through the given points.

3. $(1, 10), (2, 20)$

4. $(1, 18), (3, 162)$

5. $(2, 36), (3, 72)$

6. $(3, 375), (4, 1875)$

7. $(2, 3.6), (5, 777.6)$

8. $(2, 8), (5, 512)$

9. Describe and correct the error in determining the type of function represented by the data.

x	0	2	4	6	8
y	−2	−4	−8	−16	−32

$\times 2$ $\times 2$ $\times 2$ $\times 2$

The outputs have a common ratio of 2, but the outputs are negative, so the data does not represent a recognizable function.

In Exercises 10 and 11, determine whether the data show an exponential relationship. Then write a function that models the data.

10.

x	1	3	5	7	9
y	64	32	16	8	4

11.

x	0	10	20	30	40
y	0	15	30	45	60

12. Use a graphing calculator to find an exponential model for the data in the table.

x	2	5	6	8	9
y	7.65	25.819	38.728	87.138	130.71