6.6 Solving Exponential Equations For use with Exploration 6.6

Essential Question How can you solve an exponential equation graphically?

EXPLORATION: Solving an Exponential Equation Graphically

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use a graphing calculator to solve the exponential equation $2.5^{x-3} = 6.25$ graphically. Describe your process and explain how you determined the solution.



EXPLORATION: The Number of Solutions of an Exponential Equation

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner.

a. Use a graphing calculator to graph the equation $y = 2^x$.



- **b.** In the same viewing window, graph a linear equation (if possible) that does not intersect the graph of $y = 2^x$.
- **c.** In the same viewing window, graph a linear equation (if possible) that intersects the graph of $y = 2^x$ in more than one point.
- **d.** Is it possible for an exponential equation to have no solution? more than one solution? Explain your reasoning.

3

6.6 Solving Exponential Equations (continued)

EXPLORATION: Solving Exponential Equations Graphically

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use a graphing calculator to solve each equation.

a. $2^x = \frac{1}{2}$ **b.** $2^{x+1} = 0$ **c.** $2^x = 1$

d.
$$3^x = 9$$
 e. $3^{x-1} = 0$ **f.** $4^{2x} = \frac{1}{16}$

g.
$$2^{3x} = \frac{1}{8}$$
 h. $3^{x+2} = \frac{1}{9}$ **i.** $2^{x-2} = \frac{3}{2}x - 2$

Communicate Your Answer

- 4. How can you solve an exponential equation graphically?
- 5. A population of 30 mice is expected to double each year. The number p of mice in the population each year is given by $p = 30(2^n)$. In how many years will there be 960 mice in the population?

Name



Core Concepts

Property of Equality for Exponential Equations

Words	Two powers with the <i>same positive base b</i> , where $b \neq 1$, are equal if and only if their exponents are equal.
Numbers	If $2^x = 2^5$, then $x = 5$. If $x = 5$, then $2^x = 2^5$.
Algebra	If $b > 0$ and $b \neq 1$, then $b^x = b^y$ if and only if $x = y$.

Notes:

Worked-Out Examples

Example #1

Solve the equation. Check your solution.

$3^{9x} = 3^{7x+8}$	Check:	$3^{9x} = 3^{7x+8}$
9x = 7x + 8		$3^{9(4)} \stackrel{!}{=} 3^{7(4)+8}$
-7x - 7x		$3^{36} \stackrel{!}{=} 3^{28+8}$
2x = 8		$3^{36} = 3^{36}$
$\frac{2x}{2} = \frac{8}{2}$	1.50095	$\times 10^{17} = 1.50095 \times 10^{17}$ 🗸
x = 4		

The solution is x = 4.

6.6 Practice (continued)

Example #2

Solve the equation. Check your solution.

 $36^{-3x+3} = \left(\frac{1}{216}\right)^{x+1}$ Check: $(6^2)^{-3x+3} = \left(\frac{1}{6^3}\right)^{x+1}$ $36^{-3x+3} = \left(\frac{1}{216}\right)^{x+1}$ $6^{2(-3x+3)} = (6^{-3})^{x+1}$ $36^{-3(3)+3} \stackrel{?}{=} \left(\frac{1}{216}\right)^{3+1}$ $6^{2(-3x+3)} = 6^{-3(x+1)}$ $36^{-9+3} \stackrel{?}{=} \left(\frac{1}{216}\right)^4$ 2(-3x+3) = -3(x+1)2(-3x) + 2(3) = -3(x) - 3(1) $36^{-6} \stackrel{?}{=} \frac{1}{216^4}$ -6x + 6 = -3x - 3 $\frac{+6x}{6} = \frac{+6x}{3x-3}$ $\frac{1}{36^6} \stackrel{?}{=} \frac{1}{2,176,782,336}$ $\frac{+3}{9} = 3x^{\frac{+3}{2}}$ $\frac{1}{2,176,782,336} = \frac{1}{2,176,782,336} \checkmark$ $\frac{9}{3} = \frac{3x}{3}$ 3 = x

The solution is x = 3.

Practice A

In Exercises 1–15, solve the equation. Check your solution.

1. $3^{4x} = 3^{12}$ 2. $8^{x+5} = 8^{20}$ 3. $6^{4x-5} = 6^{2x}$ 4. $5^{6x-3} = 5^{-3+4x}$ 5. $4^{2x+11} = 1024$ 6. $8^{3-2x} = 512$ 7. $4^{7-x} = 256$ 8. $49^{x-2} = 343$ 9. $36^{6x-1} = 6^{5x}$

10.
$$9^{x-4} = 81^{3x}$$
 11. $64^{x+1} = 512^x$ **12.** $6^{2x} = 36^{2x+1}$

6.6 **Practice** (continued)

13.
$$\left(\frac{1}{7}\right)^x = 2401$$
 14. $\frac{1}{512} = 2^{3x-1}$ **15.** $25^{2-2x} = \left(\frac{1}{625}\right)^{x+1}$

In Exercises 16–21, use a graphing calculator to solve the equation.

16.
$$3^{x+3} = 9$$
 17. $\left(\frac{1}{4}\right)^{-x-1} = 64$ **18.** $-2x - 2 = -2^{-x+1}$

19.
$$2^{x+2} = 5^{x+2}$$
 20. $7^{-x+1} = 4^{x-1}$ **21.** $-\frac{1}{2}x - 3 = \left(\frac{2}{3}\right)^{2x-1}$

- **22.** You deposit \$1000 in a savings account that earns 5% annual interest compounded yearly.
 - **a.** Write an exponential equation to determine when the balance of the account will be \$1500.
 - **b.** Solve the equation.

Practice B

In Exercises 1–9, solve the equation. Check your solution.

- **1.** $3^{8x} = 3^{5x-6}$ **2.** $4^x = 2^{5x+3}$ **3.** $8^{5x} = 4^{4x+7}$ **4.** $25^{x-2} = 125^{3x+1}$ **5.** $9^{x-6} = 729^{3(x+2)}$ **6.** $4^{6(-x+2)} = 8^{-3x-4}$ **7.** $\left(\frac{1}{8}\right)^{2x+4} = 16^{4-x}$ **8.** $\left(\frac{2}{3}\right)^{x+8} = \left(\frac{3}{2}\right)^{2x-5}$ **9.** $\left(\frac{5}{4}\right)^{3x+5} = \left(\frac{16}{25}\right)^{-4x}$
- **10.** Describe and correct the error in solving the exponential equation.

Х	$\left(\frac{1}{16}\right)^{3x} = 64^{x-4}$
	$\left(4^{-2}\right)^{3x} = \left(8^2\right)^{x-4}$
	-6x = 2x - 8
	x = 1

In Exercises 11–16, use a graphing calculator to solve the equation.

- **11.** $4^{-x+2} = -\frac{1}{3}x + 5$ **12.** $\frac{1}{2}x + 3 = \left(\frac{1}{5}\right)^{2x+1}$ **13.** $6^x = 4^{-x+3}$ **14.** $5^{x-4} = 3^{-x}$ **15.** $3^{x+2} = -4^{-x+1}$ **16.** $3^{-x-5} = 2^{x+3}$
- 17. A bread dough doubles in size every hour. You begin measuring the volume of the dough 1 hour after the dough is prepared. The volume y (in cubic inches) of the dough x hours after the dough is prepared is represented by $y = 35(2^{x-1})$.

When will the volume of the dough be 4200 cubic inches?

In Exercises 18–20, solve the equation.

- **18.** $125^{x-1} = 5^{3x-2}$ **19.** $8^{2x+1} = 2^{3(2x+1)}$ **20.** $3^{8(2x-1)} = 81^{4x-2}$
- **21.** You deposit \$750 in a savings account that earns 4% annual interest compounded yearly. Write and solve an exponential equation to determine when the balance of the account will be \$1000.

In Exercises 22 and 23, use a graphing calculator to solve the equation.

22.
$$\sqrt{3} = 3^{3x-5}$$
 23. $\sqrt{2} = 2^{x-3}$