Name____ Date__________

6.8 Recursively Defined Sequences

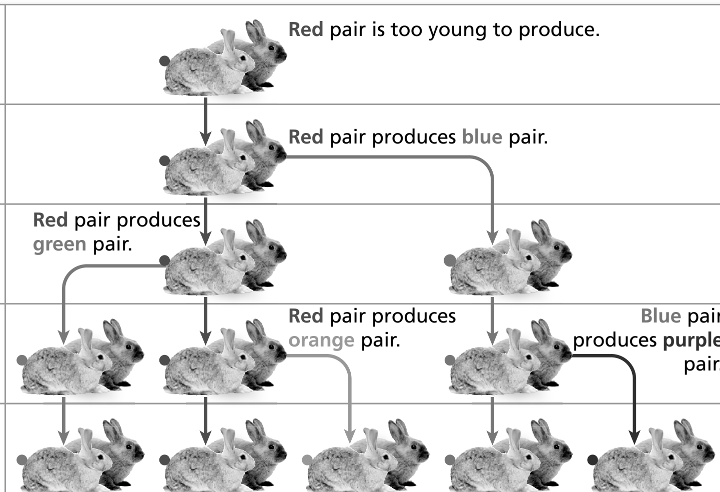
For use with Exploration 6.8

Essential Question How can you define a sequence recursively?

A **recursive rule** gives the beginning term(s) of a sequence and a *recursive equation* that tells how a_n is related to one or more preceding terms

1 EXPLORATION: Describing a Pattern

Work with a partner. Consider a hypothetical population of rabbits. Start with one breeding pair. After each month, each breeding pair produces another breeding pair. The total number of rabbits each month follows the exponential pattern 2, 4, 8, 16, 32, …. Now suppose that in the first month after each pair is born, the pair is too young to reproduce. Each pair produces another pair after it is 2 months old. Find the total number of pairs in months 6, 7, and 8.



Month		Number of pairs
1	Red pair is too young to produce.	1
2	Red pair produces blue pair.	1
3	Red pair produces green pair.	2
4	Red pair produces orange pair. Blue pair produces purple pair.	3
5		5
6		
7		
8		

2 EXPLORATION: Using a Recursive Equation

Work with a partner. Consider the following recursive equation.

$$a_n = a_{n-1} + a_{n-2}$$

Each term in the sequence is the sum of the two preceding terms.

Complete the table. Compare the results with the sequence of the number of pairs in Exploration 1.

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1						

Communicate Your Answer

3. How can you define a sequence recursively?

4. Use the Internet or some other reference to determine the mathematician who first described the sequences in Explorations 1 and 2.

Name____ Date__________

6.8 Practice
For use after Lesson 6.8

Core Concepts

Recursive Equation for an Arithmetic Sequence

$a_n = a_{n-1} + d$, where d is the common difference

Recursive Equation for a Geometric Sequence

$a_n = r \bullet a_{n-1}$, where r is the common ratio

Notes:

Worked-Out Examples

Example #1

Write the first six terms of the sequence. Then graph the sequence.

$a_1 = 2, a_n = 3a_{n-1}$

$a_1 = 2$

$a_2 = 3a_1 = 3(2) = 6$

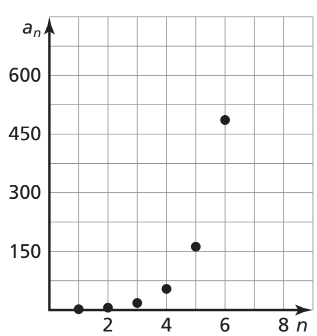
$a_3 = 3a_2 = 3(6) = 18$

$a_4 = 3a_3 = 3(18) = 54$

$a_5 = 3a_4 = 3(54) = 162$

$a_6 = 3a_5 = 3(162) = 486$

The first six terms of the sequence are 2, 6, 18, 54, 162, and 486.



Example #2

Write a recursive rule for the sequence.

243, 81, 27, 9, 3, …

Position, n	1	2	3	4	5
Term, a_n	243	81	27	9	3

$\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$

The sequence is geometric, with first term $a_1 = 243$ and common ratio $r = \frac{1}{3}$.

$a_n = r \bullet a_{n-1}$

$a_n = \frac{1}{3}a_{n-1}$

So, a recursive rule for the sequence is $a_1 = 243$, $a_n = \frac{1}{3}a_{n-1}$.

Practice A

In Exercises 1–6, write the first six terms of the sequence. Then graph the sequence.

1. $a_1 = -2$; $a_n = -2a_{n-1}$

2. $a_1 = -4$; $a_n = a_{n-1} + 3$

3. $a_1 = 4$; $a_n = 1.5a_{n-1}$

4. $a_1 = 14$; $a_n = a_{n-1} - 4$

5. $a_1 = -\frac{1}{2}$; $a_n = -2a_{n-1}$

6. $a_1 = -3$; $a_n = a_{n-1} + 2$

6.8 Practice (continued)

In Exercises 7 and 8, write a recursive rule for the sequence.

7.

n	1	2	3	4
a_n	324	108	36	12

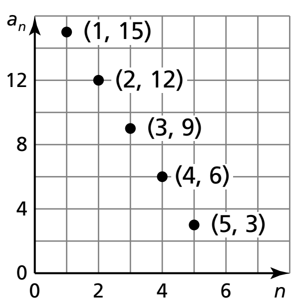
8.

n	1	2	3	4
a_n	9	14	19	24

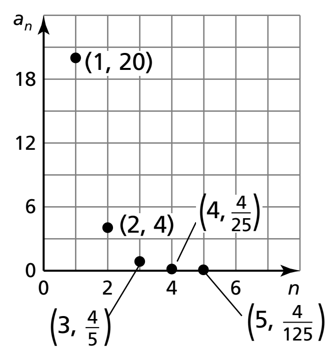
In Exercises 9–13, write a recursive rule for the sequence.

9. $3125, 625, 125, 25, \ldots$ **10.** $8, -24, 72, -216, \ldots$ **11.** $7, 13, 19, 25, \ldots$

12.



13.



In Exercises 14–16, write an explicit rule for the recursive rule.

14. $a_1 = 4; a_n = 3a_{n-1}$ **15.** $a_1 = 6; a_n = a_{n-1} + 11$ **16.** $a_1 = -1; a_n = 5\,a_{n-1}$

In Exercises 17–19, write a recursive rule for the explicit rule.

17. $a_n = 6n + 2$ **18.** $a_n = (-3)^{n-1}$ **19.** $a_n = -2n + 1$

In Exercises 20–22, write a recursive rule for the sequence. Then write the next two terms of the sequence.

20. $2, 4, 6, 10, 16, 26, \ldots$ **21.** $1, 3, -2, 5, -7, 12, \ldots$ **22.** $1, 2, 2, 4, 8, 32, \ldots$

Name ____ Date _________

Practice B

In Exercises 1 and 2, determine whether the recursive rule represents an *arithmetic sequence* or *geometric sequence*.

1. $a_1 = 5;\ a_n = 12a_{n-1}$

2. $a_1 = 6;\ a_n = a_{n-1} - 3$

In Exercises 3–6, write the first six terms of the sequence. Then graph the sequence.

3. $a_1 = 10;\ a_n = a_{n-1} - 7$

4. $a_1 = 36;\ a_n = -1.5a_{n-1}$

5. $a_1 = 120;\ a_n = \frac{1}{5}a_{n-1}$

6. $a_1 = -6;\ a_n = -3a_{n-1}$

In Exercises 7 and 8, write a recursive rule for the sequence.

7.

n	1	2	3	4
a_n	23	13	3	−7

8.

n	1	2	3	4
a_n	256	128	64	32

In Exercises 9 and 10, write an explicit rule for the recursive rule.

9. $a_1 = 8;\ a_n = -9a_{n-1}$

10. $a_1 = 5;\ a_n = a_{n-1} + 18$

In Exercises 11 and 12, write a recursive rule for the explicit rule.

11. $a_n = 1.2n + 2$

12. $a_n = -76\left(\frac{3}{2}\right)^{n-1}$

In Exercises 13 and 14, graph the first four terms of the sequence with the given description. Write a recursive rule and an explicit rule for the sequence.

13. The first term of the sequence is -2. Each term of the sequence is -5 times the preceding term.

14. The first term of the sequence is 23. Each term of the sequence is 9 less than the preceding term.

In Exercises 15 and 16, write a recursive rule for the sequence. Then write the next two terms of the sequence.

15. $4, -4, 0, -4, -4, \ldots$

16. $100, 20, 5, 4, \frac{5}{4}, \ldots$

17. Write the first five terms of the sequence $a_1 = 3;\ a_n = -a_{n-1} + 5$. Determine whether the sequence is *arithmetic*, *geometric*, *recursive*, or *none of these*. Explain your reasoning.