Name _____ Date _____

8.5 Graphing $f(x) = a(x - p)(x - q)$

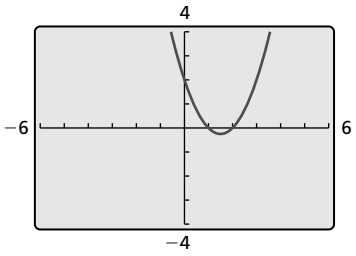
For use with Exploration 8.5

Essential Question What are some of the characteristics of the graph of $f(x) = a(x - p)(x - q)$?

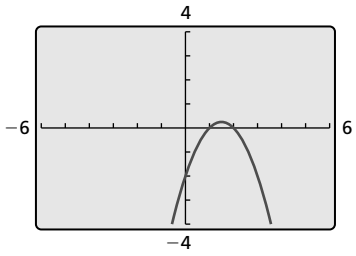
1 EXPLORATION: Using Zeros to Write Functions

Work with a partner. Each graph represents a function of the form $f(x) = (x - p)(x - q)$ or $f(x) = -(x - p)(x - q)$. Write the function represented by each graph. Explain your reasoning.

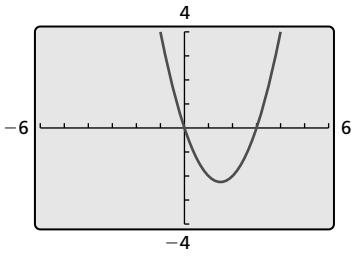
a.



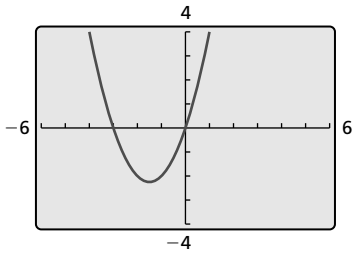
b.



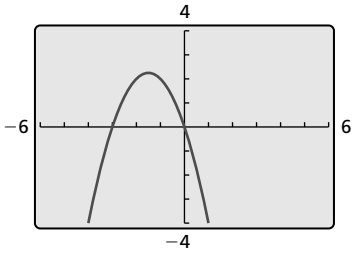
c.



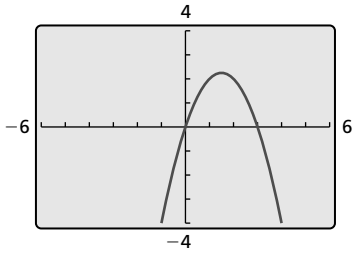
d.



e.



f.

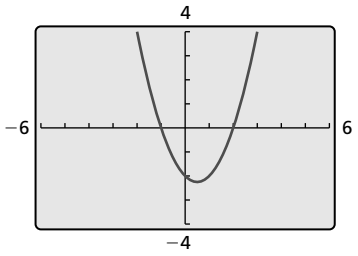


Name_____ Date _____

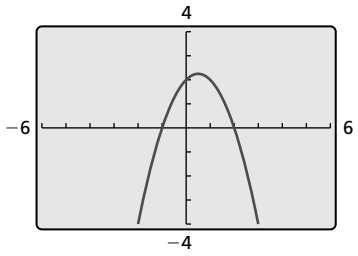
8.5 Graphing $f(x) = a(x - p)(x - q)$ (continued)

1 EXPLORATION: Using Zeros to Write Functions (continued)

g.



h.



Communicate Your Answer

2. What are some of the characteristics of the graph of $f(x) = a(x - p)(x - q)$?

3. Consider the graph of $f(x) = a(x - p)(x - q)$.

a. Does changing the sign of a change the x-intercepts? Does changing the sign of a change the y-intercept? Explain your reasoning.

b. Does changing the value of p change the x-intercepts? Does changing the value of p change the y-intercept? Explain your reasoning.

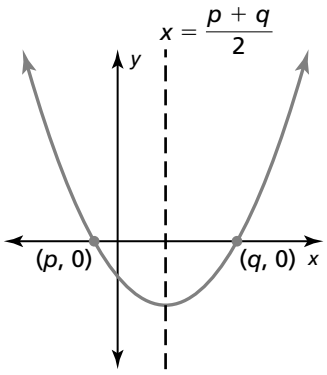
Name _____ Date _____

8.5 Practice
For use after Lesson 8.5

Core Concepts

Graphing $f(x) = a(x - p)(x - q)$

- The x-intercepts are p and q.
- The axis of symmetry is halfway between $(p, 0)$ and $(q, 0)$. So, the axis of symmetry is $x = \frac{p + q}{2}$.
- The graph opens up when $a > 0$, and the graph opens down when $a < 0$.



Notes:

Factors and Zeros

For any factor $x - n$ of a polynomial, n is a zero of the function defined by the polynomial.

Notes:

Worked-Out Examples

Example #1

Graph the quadratic function. Label the vertex, axis of symmetry, and x-intercepts. Describe the domain and range of the function.

$f(x) = (x + 4)(x + 1)$

Because the x-intercepts are $p = -4$ and $q = -1$, plot $(-4, 0)$ and $(-1, 0)$. The axis of symmetry is

$$x = \frac{p + q}{2} = \frac{-4 - 1}{2} = -\frac{5}{2}.$$

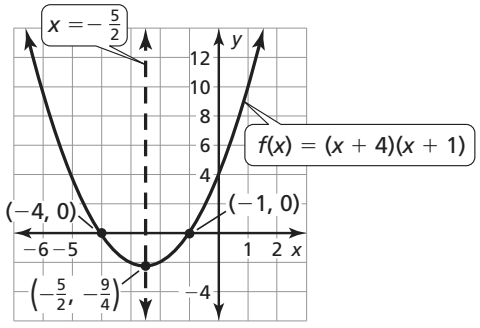
$$f(x) = (x + 4)(x + 1)$$

$$f\left(-\frac{5}{2}\right) = \left(-\frac{5}{2} + 4\right)\left(-\frac{5}{2} + 1\right)$$

$$= \left(\frac{3}{2}\right)\left(-\frac{3}{2}\right)$$

$$= -\frac{9}{4}$$

So, the vertex is $\left(-\frac{5}{2}, -\frac{9}{4}\right)$.



The domain is all real numbers. The range is $y \geq -\frac{9}{4}$.

Name _____ Date _____

8.5 Practice (continued)

Example #2

Graph the quadratic function. Label the vertex, axis of symmetry, and x-intercepts. Describe the domain and range of the function.

$q(x) = x^2 + 9x + 14$

$= (x + 7)(x + 2)$

Because the x-intercepts are $p = -7$ and $q = -2$, plot $(-7, 0)$ and $(-2, 0)$. The axis of symmetry is

$$x = \frac{p + q}{2} = \frac{-7 - 2}{2} = -\frac{9}{2}.$$

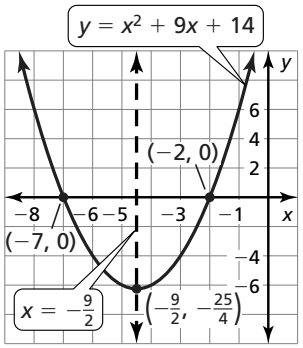
$$q(x) = x^2 + 9x + 14$$

$$q\left(-\frac{9}{2}\right) = \left(-\frac{9}{2}\right)^2 + 9\left(-\frac{9}{2}\right) + 14$$

$$= \frac{81}{4} - \frac{81}{2} + 14$$

$$= -\frac{81}{4} + 14$$

$$= -\frac{25}{4}$$



The domain is all real numbers.

The range is $y \geq -\frac{25}{4}$.

Practice A

In Exercises 1 and 2, find the *x*-intercepts and axis of symmetry of the graph of the function.

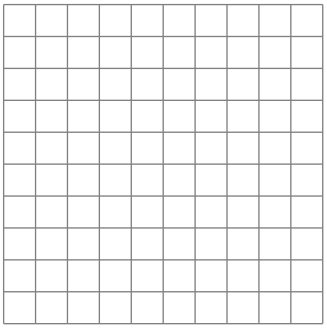
1. $y = (x + 2)(x - 4)$

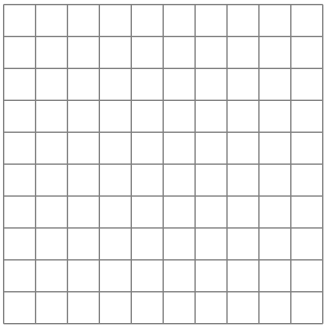
2. $y = -3(x - 2)(x - 3)$

In Exercises 3–6, graph the quadratic function. Label the vertex, axis of symmetry, and *x*-intercepts. Describe the domain and range of the function.

3. $m(x) = (x + 5)(x + 1)$

4. $y = -4(x - 3)(x - 1)$

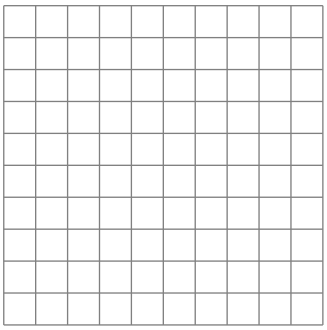




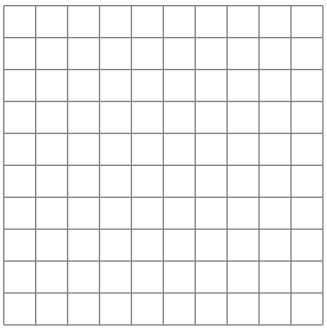
Name _____ Date _____

8.5 Practice (continued)

5. $y = x^2 - 4$



6. $f(x) = x^2 + 2x - 15$



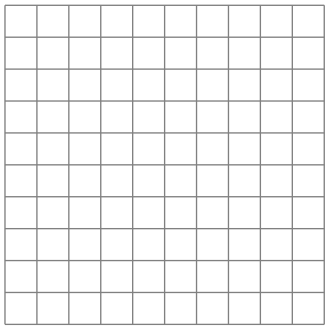
In Exercises 7 and 8, find the zero(s) of the function.

7. $y = 6x^2 - 6$

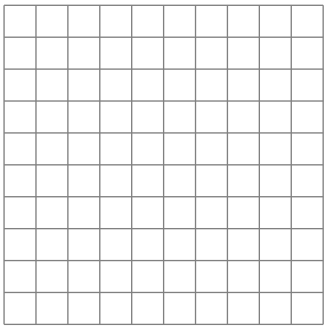
8. $y = x^2 + 9x + 20$

In Exercises 9–12, use zeros to graph the function.

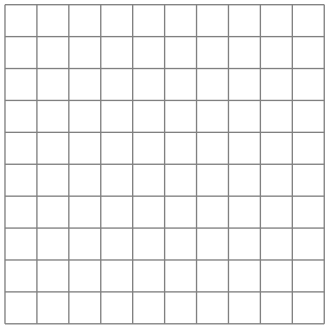
9. $f(x) = x^2 - 3x - 10$



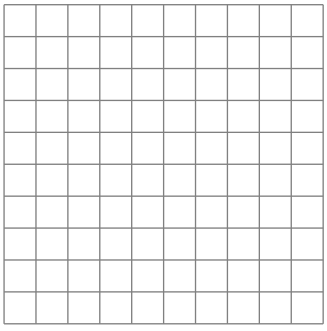
10. $f(x) = -2(x + 3)(x - 1)$



11. $f(x) = x^2 - 9$



12. $f(x) = 2x^2 - 12x + 10$



Name _____ Date _____

Practice B

In Exercises 1 and 2, find the *x*-intercepts and axis of symmetry of the graph of the function.

1. $f(x) = -\frac{1}{3}x(x + 5)$

2. $g(x) = 9(x + 6)(x - 4)$

In Exercises 3–6, graph the quadratic function. Label the vertex, axis of symmetry, and *x*-intercepts. Describe the domain and range of the function.

3. $f(x) = 4(x + 3)(x + 2)$

4. $y = -3(x - 4)(x + 2)$

5. $p(x) = x^2 - 7x + 12$

6. $y = 2x^2 + 20x + 42$

In Exercises 7–10, find the zero(s) of the function.

7. $f(x) = \frac{2}{3}(x + 8)(x - 5)$

8. $g(x) = 3x^2 + 13x + 4$

9. $y = (x^2 - 25)(x + 7)$

10. $y = x^3 - 81x$

In Exercises 11–14, use zeros to graph the function.

11. $f(x) = -2(x - 5)(x - 3)$

12. $g(x) = x^2 + 2x - 24$

13. $y = -4x^2 - 16x + 20$

14. $f(x) = 3x^2 - 12$

In Exercises 15–19, write a quadratic function in standard form whose graph satisfies the given conditions.

15. vertex: $(6, -2)$

16. x-intercepts: 5 and -8

17. passes through $(-4, 0)$, $(2, 0)$, and $(0, -4)$

18. y decreases as x increases when $x < 1$; y increases as x increases when $x > 1$

19. range: $y \leq 6$

20. The cross section of a satellite dish can be modeled by the function $y = \frac{1}{6}(x^2 - 9)$, where x and y are measured in feet. The x-axis represents the top of the opening of the dish.

a. How wide is the satellite dish?

b. How deep is the satellite dish?