Name ____ Date __________

8.6 Transformations of Quadratic Functions

For use with Exploration 8.6

Essential Question How do the constants a, h, and k affect the graph of the quadratic function $g(x) = a(x - h)^2 + k$?

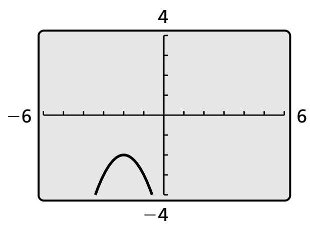
1 EXPLORATION: Identifying Graphs of Quadratic Functions

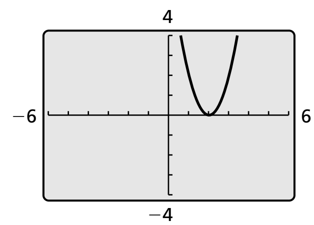
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Match each quadratic function with its graph. Explain your reasoning. Then use a graphing calculator to verify that your answer is correct.

a. $g(x) = -(x - 2)^2$ **b.** $g(x) = (x - 2)^2 + 2$ **c.** $g(x) = -(x + 2)^2 - 2$

d. $g(x) = 0.5(x - 2)^2 - 2$ **e.** $g(x) = 2(x - 2)^2$ **f.** $g(x) = -(x + 2)^2 + 2$

A.


B.


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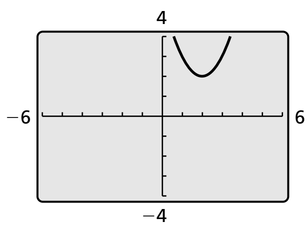

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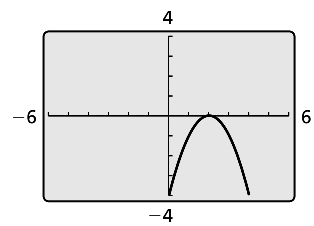
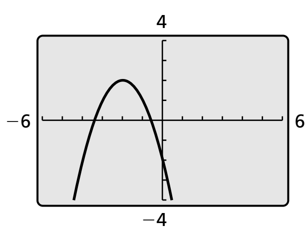


E.


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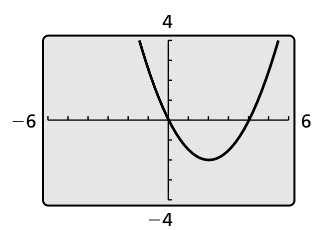
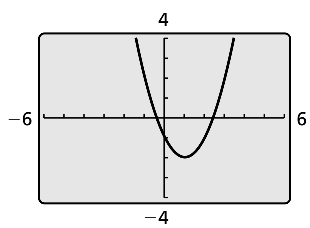


Communicate Your Answer

2. How do the constants a, h, and k affect the graph of the quadratic function $g(x) = a(x - h)^2 + k$?

3. Write the equation of the quadratic function whose graph is shown. Explain your reasoning. Then use a graphing calculator to verify that your equation is correct.



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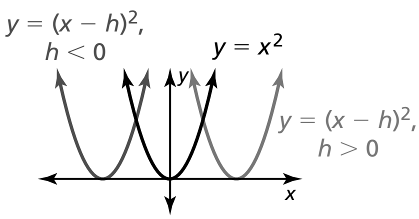
8.6 Practice
For use after Lesson 8.6

Core Concepts

Horizontal Translations

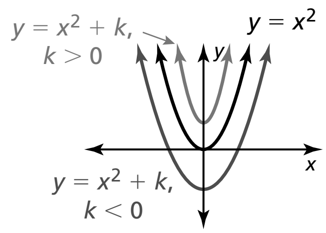
$$f(x) = x^2$$
$$f(x - h) = (x - h)^2$$



- shifts left when $h < 0$
- shifts right when $h > 0$

Vertical Translations

$$f(x) = x^2$$
$$f(x) + k = x^2 + k$$

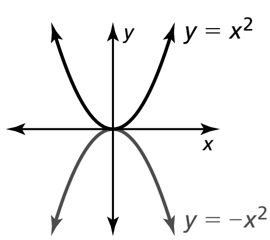


- shifts down when $k < 0$
- shifts up when $k > 0$

Notes:

Reflections in the *x*-Axis

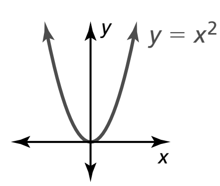
$$f(x) = x^2$$
$$-f(x) = -(x^2) = -x^2$$



flips over the x-axis

Reflections in the *y*-Axis

$$f(x) = x^2$$
$$f(-x) = (-x)^2 = x^2$$



$y = x^2$ is its own reflection in the y-axis.

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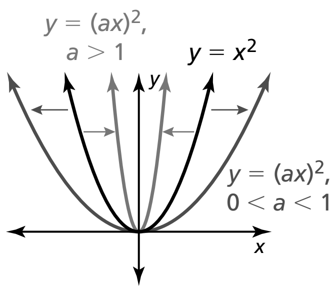
8.6 Practice (continued)

Horizontal Stretches and Shrinks

$$f(x) = x^2$$

$$f(ax) = (ax)^2$$

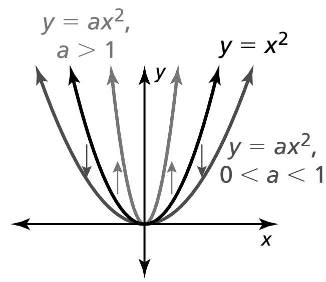


- horizontal stretch (away from y-axis) when $0 < a < 1$
- horizontal shrink (toward y-axis) when $a > 1$

Vertical Stretches and Shrinks

$$f(x) = x^2$$

$$a \bullet f(x) = ax^2$$



- vertical stretch (away from x-axis) when $a > 1$
- vertical shrink (toward x-axis) when $0 < a < 1$

Notes:

Worked-Out Examples

Example #1

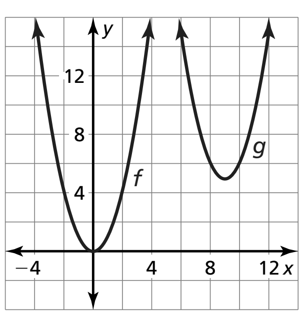
Describe the transformation of f(x) = x² represented by g. Then graph each function.

$g(x) = (x - 9)^2 + 5$

Notice that the function is of the form $g(x) = (x - h)^2 + k$.
Rewrite the function to identify h and k.

$g(x) = (x - (9))^2 + (5)$

Because $h = -9$ and $k = 5$, the graph of g is a translation 9 units right and 5 units up of the graph of f.



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8.6 **Practice (continued)**

Example #2

Describe the transformation of the graph of the parent quadratic function. Then identify the vertex.

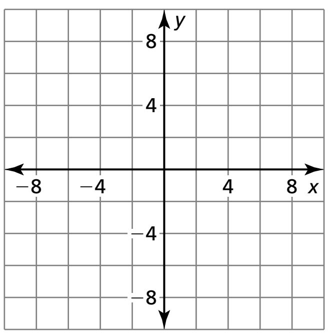
$f(x) = \frac{1}{2}(x - 1)^2$

The transformation is a horizontal translation to the right 1 unit, followed by a vertical shrink by a factor of $\frac{1}{2}$; The vertex is (1, 0).

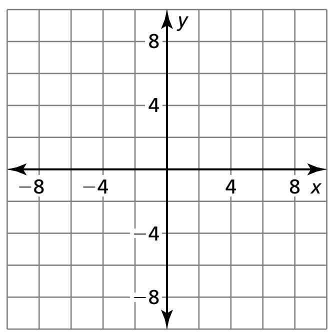
Practice A

In Exercises 1–6, describe the transformation of $f(x) = x^2$ represented by g. Then graph the function.

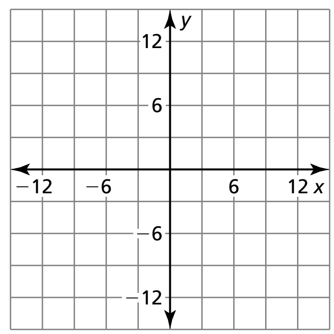
1. $g(x) = x^2 + 4$



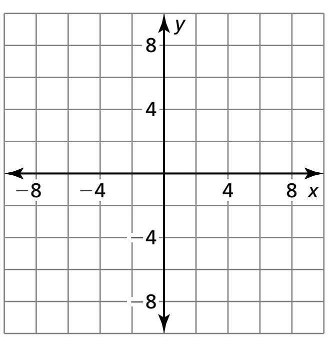
2. $g(x) = (x - 1)^2 - 3$



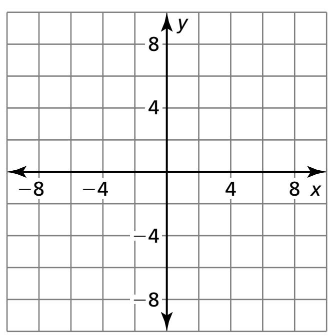
3. $g(x) = -(x + 9)^2$



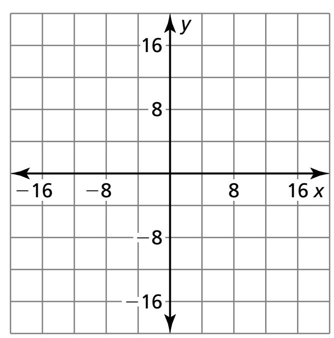
4. $g(x) = x^2 - 7$



5. $g(x) = \frac{1}{3}x^2 - 6$



6. $g(x) = (-4x)^2$



7. Consider the function $f(x) = -10(x - 5)^2 + 7$. Describe the transformation of the graph of the parent quadratic function. Then identify the vertex.

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Practice B

In Exercises 1–6, describe the transformation of $f(x) = x^2$ represented by g. Then graph each function.

1. $g(x) = x^2 + 3$

2. $g(x) = (x + 5)^2$

3. $g(x) = (x + 6)^2 - 4$

4. $g(x) = (x - 1)^2 + 5$

5. $g(x) = (x - 4)^2 + 3$

6. $g(x) = (x + 8)^2 - 2$

In Exercises 7–9, describe the transformation of $f(x) = x^2$ represented by g. Then graph each function.

7. $g(x) = -\left(\frac{1}{2}x\right)^2$

8. $g(x) = \frac{1}{3}x^2 + 2$

9. $g(x) = \frac{1}{3}(x + 1)^2$

In Exercises 10 and 11, describe the transformation of the graph of the parent quadratic function. Then identify the vertex.

10. $f(x) = -3(x + 6)^2 - 4$

11. $f(x) = \frac{1}{3}(x - 2)^2 + 1$

In Exercises 12 and 13, write a rule for *g* described by the transformations of the graph of *f*. Then identify the vertex.

12. $f(x) = x^2$; vertical shrink by a factor of $\frac{1}{2}$ and a reflection in the y-axis, followed by a translation 2 units left

13. $f(x) = (x + 4)^2 + 2$; horizontal shrink by a factor of $\frac{1}{3}$ and a translation 2 units up, followed by a reflection in the x-axis

14. Justify each step in writing a function g based on the transformations of $f(x) = 4x^2 - 3x$.

translation 3 units up followed by a reflection in the y-axis

$h(x) = f(x) + 3$	
$= 4x^2 - 3x + 3$	
$g(x) = h(-x)$	
$= 4x^2 + 3x + 3$	