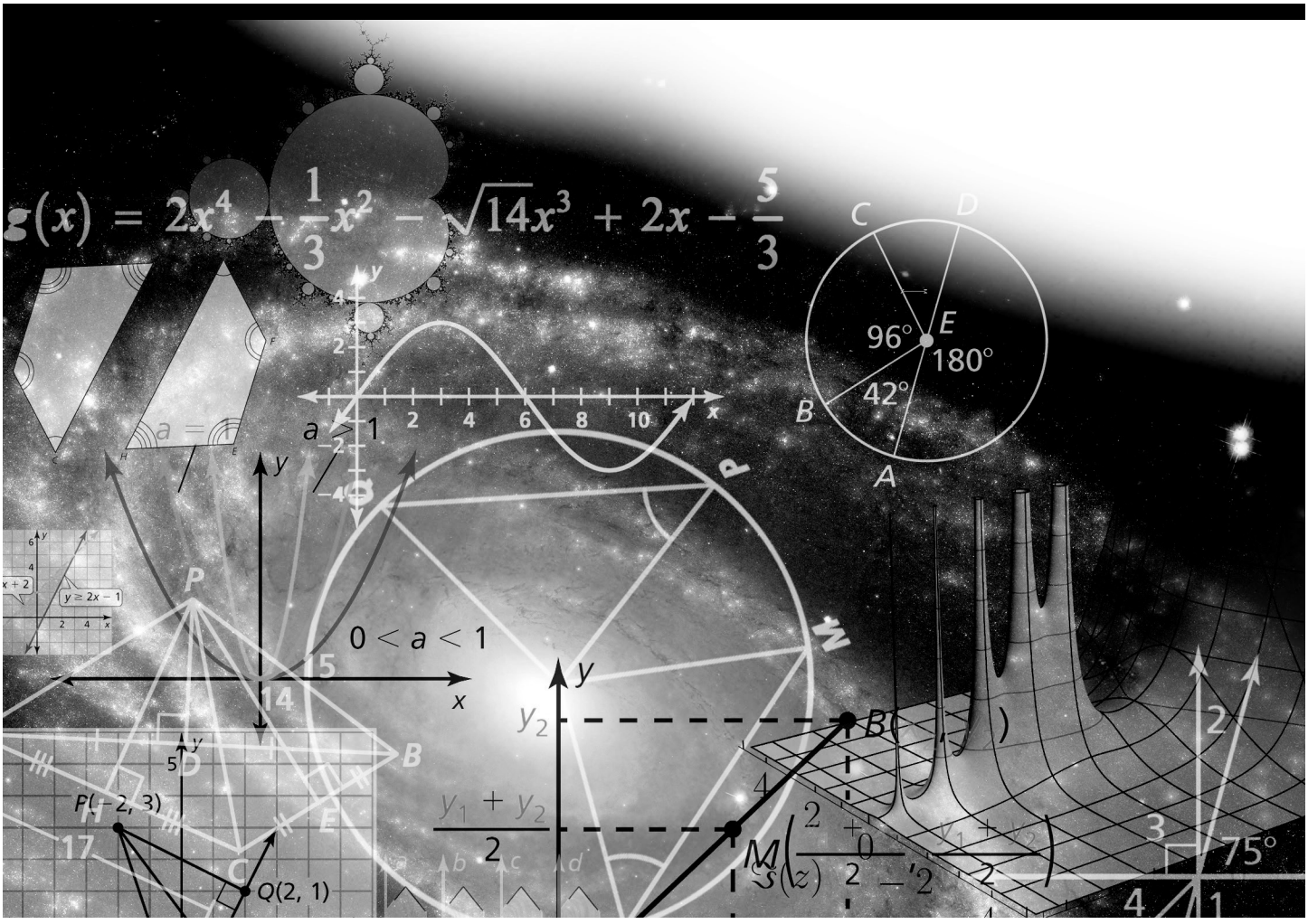
CHAPTER 2

Solving Quadratic Equations



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Chapter 2 Maintaining Mathematical Proficiency

Factor the trinomial.

1. $x^2 - 6x + 9$

2. $x^2 + 4x + 4$

3. $x^2 - 14x + 49$

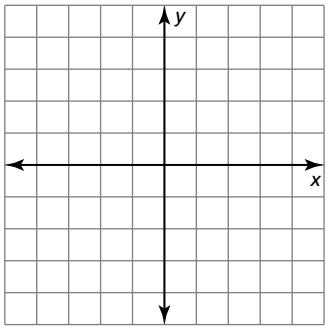
4. $x^2 + 22x + 121$

5. $x^2 - 24x + 144$

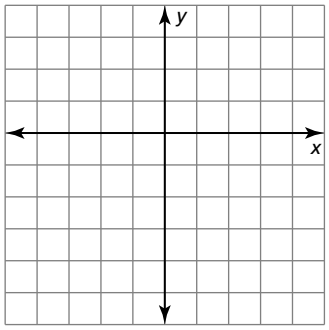
6. $x^2 + 26x + 169$

Solve the system of linear equations by graphing.

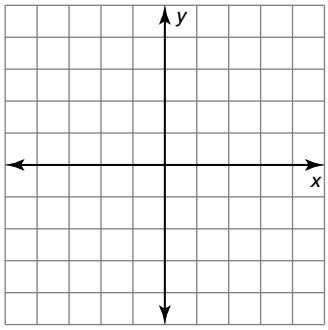
7. $y = 2x - 1$
$y = -3x + 9$



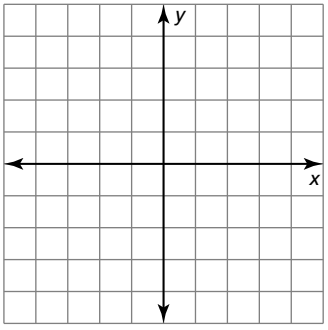
8. $y = -\frac{1}{2}x - 1$
$y = \frac{1}{4}x - 4$



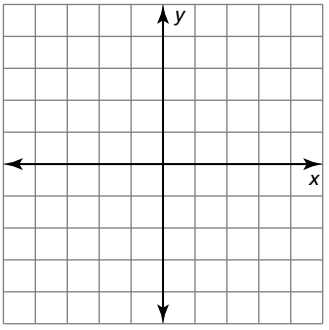
9. $y = 2x + 3$
$y = -3x - 2$



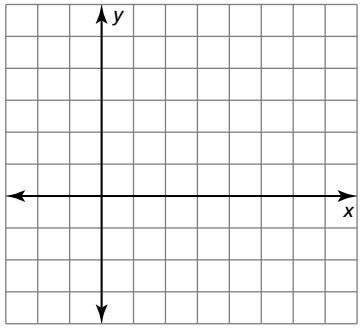
10. $y = x + 3$
$y = -\frac{1}{3}x - 1$



11. $y = x + 1$
$y = 3x - 1$



12. $y = 2x - 3$
$y = x + 1$



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2.1 Adding, Subtracting, and Multiplying Polynomials

For use with Exploration 2.1

Essential Question How can you cube a binomial?

1 EXPLORATION: Cubing Binomials

Work with a partner. Find each product. Show your steps.

a. $(x + 1)^3 = (x + 1)(x + 1)^2$ Rewrite as a product of first and second powers.

$= (x + 1)$ _____ Multiply second power.

$=$ _____ Multiply binomial and trinomial.

$=$ _____ Write in standard form, $ax^3 + bx^2 + cx + d$.

b. $(a + b)^3 = (a + b)(a + b)^2$ Rewrite as a product of first and second powers.

$= (a + b)$ _____ Multiply second power.

$=$ _____ Multiply binomial and trinomial.

$=$ _____ Write in standard form.

c. $(x - 1)^3 = (x - 1)(x - 1)^2$ Rewrite as a product of first and second powers.

$= (x - 1)$ _____ Multiply second power.

$=$ _____ Multiply binomial and trinomial.

$=$ _____ Write in standard form.

d. $(a - b)^3 = (a - b)(a - b)^2$ Rewrite as a product of first and second powers.

$= (a - b)$ _____ Multiply second power.

$=$ _____ Multiply binomial and trinomial.

$=$ _____ Write in standard form.

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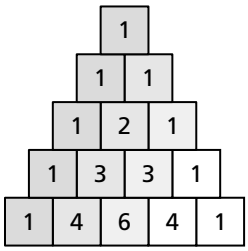
2.1 Adding, Subtracting, and Multiplying Polynomials (continued)

2 EXPLORATION: Generalizing Patterns for Cubing a Binomial

Work with a partner.

a. Use the results of Exploration 1 to describe a pattern for the coefficients of the terms when you expand the cube of a binomial. How is your pattern related to Pascal's Triangle, shown at the right?



b. Use the results of Exploration 1 to describe a pattern for the exponents of the terms in the expansion of a cube of a binomial.

c. Explain how you can use the patterns you described in parts (a) and (b) to find the product $(2x - 3)^3$. Then find this product.

Communicate Your Answer

3. How can you cube a binomial?

4. Find each product.

a. $(x + 2)^3$ **b.** $(x - 2)^3$ **c.** $(2x - 3)^3$

d. $(x - 3)^3$ **e.** $(-2x + 3)^3$ **f.** $(3x - 5)^3$

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2.1 Practice

For use after Lesson 2.1

Core Concepts

Special Product Patterns

Sum and Difference	Example
$(a + b)(a - b) = a^2 - b^2$	$(x + 3)(x - 3) = x^2 - 9$

Square of a Binomial	Example
$(a + b)^2 = a^2 + 2ab + b^2$	$(y + 4)^2 = y^2 + 8y + 16$
$(a - b)^2 = a^2 - 2ab + b^2$	$(2t - 5)^2 = 4t^2 - 20t + 25$

Cube of a Binomial	Example
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	$(z + 3)^3 = z^3 + 9z^2 + 27z + 27$
$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$	$(m - 2)^3 = m^3 - 6m^2 + 12m - 8$

Notes:

Pascal's Triangle

In Pascal's Triangle, the first and last numbers in each row are 1. Every number other than 1 is the sum of the closest two numbers in the row directly above it. The numbers in Pascal's Triangle are the same numbers that are the coefficients of binomial expansions, as shown in the first six rows.

	n	$(a + b)^n$	Binomial Expansion	Pascal's Triangle
0th row	0	$(a + b)^0 =$	1	1
1st row	1	$(a + b)^1 =$	$1a + 1b$	1 1
2nd row	2	$(a + b)^2 =$	$1a^2 + 2ab + 1b^2$	1 2 1
3rd row	3	$(a + b)^3 =$	$1a^3 + 3a^2b + 3ab^2 + 1b^3$	1 3 3 1
4th row	4	$(a + b)^4 =$	$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$	1 4 6 4 1
5th row	5	$(a + b)^5 =$	$1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$	1 5 10 10 5 1

Notes:

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2.1 Practice (continued)

The Binomial Theorem

For any positive integer n, the binomial expansion of $(a + b)^n$ is

$$(a + b)^n = {}_nC_0a^nb^0 + {}_nC_1a^{n-1}b^1 + {}_nC_2a^{n-2}b^2 + \cdots + {}_nC_na^0b^n.$$

Notice that each term in the expansion of $(a + b)^n$ has the form ${}_nC_ra^{n-r}b^r$, where r is an integer from 0 to n.

Notes:

Worked-Out Examples

Example #1

Find the difference.

$$\begin{aligned}(4x^5 - 7x^3 - 9x^2 + 18) - (14x^5 - 8x^4 + 11x^2 + x)\\ &= 4x^5 - 7x^3 - 9x^2 + 18 - 14x^5 + 8x^4 - 11x^2 - x\\ &= -10x^5 + 8x^4 - 7x^3 - 20x^2 - x + 18\end{aligned}$$

Example #2

Find the product.

$$\begin{aligned}(5x^2 - 4x + 6)(-2x + 3)\\ &= (5x^2 - 4x + 6)(-2x) + (5x^2 - 4x + 6)3\\ &= -10x^3 + 8x^2 - 12x + 15x^2 - 12x + 18\\ &= -10x^3 + 23x^2 - 24x + 18\end{aligned}$$

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2.1 Practice (continued)

Practice A

In Exercises 1–3, find the sum or difference.

1. $\left(-4x^2 - 6x + 18\right) + \left(-x^2 + 7x + 8\right)$

2. $\left(6x^2 - 12x + 48\right) - \left(-x^2 + 24x - 63\right)$

3. $\left(-11x^4 - x^3 - 3x^2 + 10x - 2\right) - \left(-11x^4 + 5x^2 - 7x + 13\right)$

In Exercises 4–9, find the product.

4. $\left(x^4 - 10x^2 + 25\right)\left(3x^2 - 6x - 1\right)$

5. $(2x - 3)(6 - x)(4 - 5x)$

6. $(3y - 8)(3y + 8)$

7. $(2v - 1)^3$

In Exercises 8 and 9, use Pascal's Triangle to expand the binomial.

8. $(4t - 2)^4$

9. $(g + 6)^5$

10. Use the Binomial Theorem to write the binomial expansion of $\left(2x^4 + y^3\right)^3$.

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Practice B

In Exercises 1 and 2, find the sum.

1. $(8x^7 - 6x^5 + 4x^3 - 6x) + (15x^6 + 4x^5 - 3x^3 + 2)$

2. $(8x^4 - 2x^3 + 9x^2 + 7x + 14) + (6x^4 - 5x^3 - 9x^2 - 11x - 9)$

In Exercises 3 and 4, find the difference.

3. $(9x^5 + 5x^4 - 9x^2 + 10x) - (12x^5 + 2x^4 - x^2 - 9)$

4. $(12x^4 - 6x^2 + 2x + 14) - (3x^4 - 5x^3 + 9x + 3)$

In Exercises 5–8, find the product.

5. $(x^2 - 7x - 2)(x^2 - 3x - 6)$

6. $(2x^2 + 3x - 1)(-5x^2 - 2x + 4)$

7. $(4x^2 - 3x + 6)(x^2 - 2x + 2)$

8. $(3x^2 - 6x - 5)(x^4 + 2x^2 + 5x)$

9. Describe and correct the error in performing the operation.

$\times \quad 4x^2(3x^4 - 2x^3 + 7) = 12x^8 - 8x^6 + 28x^2$

In Exercises 10–13, find the product of the binomials.

10. $(x - 3)(2x + 2)(3x - 1)$

11. $(2x + 3)(x - 5)(4x + 1)$

12. $(2x - 1)(3 - 2x)(4x + 5)$

13. $(5 - 2x)(2 - x)(4x + 3)$

In Exercises 14–16, find the product.

14. $(3x + 5)(3x - 5)$

15. $(6t + 7)^2$

16. $(pq + 2)^2$

17. A rectangular pool has a level floor. The length of the pool is $(3x - 1)$ feet, the width of the pool is $(x + 6)$ feet, and the depth of the pool is $(x + 6)$ feet.

a. Write an expression for the volume of the pool as a product of binomials.

b. Write an expression for the volume of the pool as a polynomial in standard form.

18. Use Pascal's Triangle to expand $(2m - 5)^5$.

19. Use the Binomial Theorem to write the binomial expansion of $(3s + t)^3$.