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2.2 Properties of Radicals

For use with Exploration 2.2

Essential Question How can you multiply and divide square roots?

1 EXPLORATION: Operations with Square Roots

Work with a partner. For each operation with square roots, compare the results obtained using the two indicated orders of operations. What can you conclude?

a. Square Roots and Addition

Is $\sqrt{36} + \sqrt{64}$ equal to $\sqrt{36 + 64}$?

In general, is $\sqrt{a} + \sqrt{b}$ equal to $\sqrt{a + b}$? Explain your reasoning.

b. Square Roots and Multiplication

Is $\sqrt{4} \bullet \sqrt{9}$ equal to $\sqrt{4 \bullet 9}$?

In general, is $\sqrt{a} \bullet \sqrt{b}$ equal to $\sqrt{a \bullet b}$? Explain your reasoning.

c. Square Roots and Subtraction

Is $\sqrt{64} - \sqrt{36}$ equal to $\sqrt{64 - 36}$?

In general, is $\sqrt{a} - \sqrt{b}$ equal to $\sqrt{a - b}$? Explain your reasoning.

d. Square Roots and Division

Is $\dfrac{\sqrt{100}}{\sqrt{4}}$ equal to $\sqrt{\dfrac{100}{4}}$?

In general, is $\dfrac{\sqrt{a}}{\sqrt{b}}$ equal to $\sqrt{\dfrac{a}{b}}$? Explain your reasoning.

2 EXPLORATION: Writing Counterexamples

Work with a partner. A **counterexample** is an example that proves that a general statement is *not* true. For each general statement in Exploration 1 that is not true, write a counterexample different from the example given.

Communicate Your Answer

3. How can you multiply and divide square roots?

4. Give an example of multiplying square roots and an example of dividing square roots that are different from the examples in Exploration 1.

5. Write an algebraic rule for each operation.

a. the product of square roots

b. the quotient of square roots

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2.2 Practice
For use after Lesson 2.2

Core Concepts

Product Property of Square Roots

Words The square root of a product equals the product of the square roots of the factors.

Numbers $\sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

Algebra $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where $a, b \geq 0$

Notes:

Quotient Property of Square Roots

Words The square root of a quotient equals the quotient of the square roots of the numerator and denominator.

Numbers $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$ **Algebra** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$

Notes:

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2.2 **Practice (continued)**

Worked-Out Examples

Example #1

Simplify the expression.

$$\begin{aligned}\sqrt{48n^5} &= \sqrt{16 \bullet 3 \bullet n^4 \bullet n}\\ &= \sqrt{16} \bullet \sqrt{3} \bullet \sqrt{n^4} \bullet \sqrt{n}\\ &= 4 \bullet \sqrt{3} \bullet n^2 \bullet \sqrt{n}\\ &= 4 \bullet n^2 \bullet \sqrt{3} \bullet \sqrt{n}\\ &= 4n^2\sqrt{3n}\end{aligned}$$

Example #2

Simplify the expression.

$$\begin{aligned}\frac{\sqrt{5}}{\sqrt{48}} &= \frac{\sqrt{5}}{\sqrt{16 \bullet 3}}\\ &= \frac{\sqrt{5}}{\sqrt{16} \bullet \sqrt{3}}\\ &= \frac{\sqrt{5}}{4 \bullet \sqrt{3}}\\ &= \frac{\sqrt{5}}{4 \bullet \sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}}\\ &= \frac{\sqrt{15}}{4 \bullet \sqrt{9}}\\ &= \frac{\sqrt{15}}{4 \cdot 3}\\ &= \frac{\sqrt{15}}{12}\end{aligned}$$

Practice A

In Exercises 1–12, simplify the expression.

1. $\sqrt{24}$ **2.** $-\sqrt{48}$ **3.** $\sqrt{162g^6}$ **4.** $-\sqrt{512h^7}$

5. $\sqrt{\frac{25}{64}}$ **6.** $-\sqrt{\frac{6}{49}}$ **7.** $-\sqrt{\frac{196}{r^4}}$ **8.** $\sqrt{\frac{49x^3}{64y^2}}$

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2.2 Practice (continued)

In Exercises 13–20, simplify the expression.

9. $\sqrt[3]{-135}$

10. $\sqrt[3]{729}$

11. $-\sqrt[3]{-192x^5}$

12. $\sqrt[3]{\dfrac{12a^6}{512b^4}}$

13. $\dfrac{\sqrt{15}}{\sqrt{500}}$

14. $\sqrt{\dfrac{8}{100}}$

15. $\dfrac{\sqrt{3x^2y^3}}{\sqrt{80xy^3}}$

16. $\dfrac{8}{\sqrt[3]{16}}$

17. $\dfrac{5}{-3 - 3\sqrt{3}}$

18. $\dfrac{3}{4 + 4\sqrt{5}}$

19. $\dfrac{4}{\sqrt{2} - 5\sqrt{3}}$

20. $\dfrac{\sqrt{5}}{\sqrt{3} + \sqrt{5}}$

21. The ratio of the length to the width of a *golden rectangle* is $\left(1 + \sqrt{5}\right) : 2$. The length of a golden rectangle is 62 meters. What is the width? Round your answer to the nearest meter.

In Exercises 22–27, simplify the expression.

22. $3\sqrt{8} + 3\sqrt{2}$

23. $2\sqrt{18} - 2\sqrt{20} - 2\sqrt{5}$

24. $3\sqrt{12} + 3\sqrt{18} + 2\sqrt{27}$

25. $2\sqrt{5}\left(\sqrt{6} + 2\right)$

26. $\left(\sqrt{7} - \sqrt{3}\right)\left(\sqrt{7} + \sqrt{3}\right)$

27. $\sqrt[3]{2}\left(\sqrt[3]{108} - \sqrt[3]{135}\right)$

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Practice B

In Exercises 1–9, simplify the expression.

1. $\sqrt{54}$

2. $\sqrt{25y^2}$

3. $-\sqrt{18n^3}$

4. $\sqrt{\dfrac{29}{100}}$

5. $\sqrt{\dfrac{p^3}{49}}$

6. $\sqrt{\dfrac{36}{9x^2}}$

7. $\sqrt[3]{32q^2}$

8. $\sqrt[3]{\dfrac{9d}{-8}}$

9. $-\sqrt[3]{\dfrac{60x^2}{729y^3}}$

10. Describe and correct the error in simplifying the expression.

$$\times \quad \sqrt{\frac{30}{25}} = \sqrt{\frac{6}{5}} = \frac{\sqrt{6}}{\sqrt{5}}$$

In Exercises 11–13, write a factor that you can use to rationalize the denominator of the expression.

11. $\dfrac{2}{\sqrt{7y}}$

12. $\dfrac{8}{\sqrt[3]{k^2}}$

13. $\dfrac{2}{3 - \sqrt{6}}$

In Exercises 14–22, simplify the expression.

14. $\dfrac{4}{\sqrt{3}}$

15. $\dfrac{\sqrt{2}}{\sqrt{45}}$

16. $\dfrac{1}{\sqrt{6t}}$

17. $\sqrt{\dfrac{5h^2}{7}}$

18. $\dfrac{\sqrt{27}}{\sqrt{2d^3}}$

19. $\dfrac{25}{\sqrt[3]{4}}$

20. $\dfrac{5}{7 - \sqrt{2}}$

21. $\dfrac{\sqrt{3}}{8 + \sqrt{7}}$

22. $\dfrac{\sqrt{5}}{\sqrt{5} - \sqrt{7}}$

23. Use the special product pattern $(a - b)(a^2 + ab + b^2) = a^3 - b^3$ to simplify the expression $\dfrac{3}{\sqrt[3]{x} - 1}$.