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2.4 Solving Quadratic Equations Using the Quadratic Formula

For use with Exploration 2.4

Essential Question How can you derive a formula that can be used to write the solutions of any quadratic equation in standard form?

1 EXPLORATION: Deriving the Quadratic Formula

Work with a partner. The following steps show a method of solving $ax^2 + bx + c = 0$. Explain what was done in each step.

$ax^2 + bx + c = 0$ 1. Write the equation.

$4a^2x^2 + 4abx + 4ac = 0$ 2. _____

$4a^2x^2 + 4abx + 4ac + b^2 = b^2$ 3. _____

$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$ 4. _____

$(2ax + b)^2 = b^2 - 4ac$ 5. _____

$2ax + b = \pm\sqrt{b^2 - 4ac}$ 6. _____

$2ax = -b \pm \sqrt{b^2 - 4ac}$ 7. _____

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 8. _____

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2 EXPLORATION: Deriving the Quadratic Formula by Completing the Square

Work with a partner.

a. Solve $ax^2 + bx + c = 0$ by completing the square. (*Hint:* Subtract c from each side, divide each side by a, and then proceed by completing the square.)

b. Compare this method with the method in Exploration 1. Explain why you think $4a$ and b^2 were chosen in Steps 2 and 3 of Exploration 1.

Communicate Your Answer

3. How can you derive a formula that can be used to write the solutions of any quadratic equation in standard form?

4. Use the Quadratic Formula to solve each quadratic equation.

a. $x^2 + 2x - 3 = 0$ **b.** $x^2 - 4x + 4 = 0$ **c.** $x^2 + 4x + 5 = 0$

5. Use the Internet to research *imaginary numbers*. How are they related to quadratic equations?

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2.4 Practice
For use after Lesson 2.4

Core Concepts

Quadratic Formula

The real solutions of the quadratic equation $ax^2 + bx + c = 0$ are

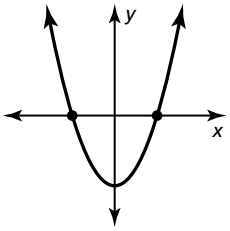
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

where $a \neq 0$ and $b^2 - 4ac \geq 0$.

Notes:

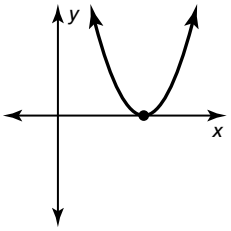
Interpreting the Discriminant

$b^2 - 4ac > 0$



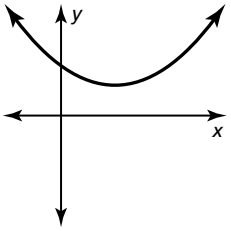
- two real solutions
- two x-intercepts

$b^2 - 4ac = 0$



- one real solution
- one x-intercept

$b^2 - 4ac < 0$



- no real solutions
- no x-intercepts

Notes:

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2.4 Practice (continued)

Methods for Solving Quadratic Equations

Method	Advantages	Disadvantages
Factoring (*Lessons 2.5–2.8*)	• Straightforward when the equation can be factored easily	• Some equations are not factorable.
Graphing (*Lesson 4.2*)	• Can easily see the number of solutions • Use when approximate solutions are sufficient. • Can use a graphing calculator	• May not give exact solutions
Using Square Roots (*Lesson 4.3*)	• Used to solve equations of the form $x^2 = d$.	• Can only be used for certain equations
Completing the Square (*Lesson 4.4*)	• Best used when $a = 1$ and b is even	• May involve difficult calculations
Quadratic Formula (*Lesson 4.5*)	• Can be used for any quadratic equation • Gives exact solutions	• Takes time to do calculations

Notes:

Worked-Out Examples

Example #1

Solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary.

$$6x^2 - 13x = -6$$
$$6x^2 - 13x + 6 = -6 + 6$$
$$6x^2 - 13x + 6 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-13) \pm \sqrt{(-13)^2 - 4(6)(6)}}{2(6)}$$
$$= \frac{13 \pm \sqrt{169 - 144}}{12}$$
$$= \frac{13 \pm \sqrt{25}}{12}$$
$$= \frac{13 \pm 5}{12}$$

The solutions are $x = \frac{13 + 5}{12} = \frac{18}{12} = \frac{3}{2}$ and $x = \frac{13 - 5}{12} = \frac{8}{12} = \frac{2}{3}$.

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2.4 Practice (continued)

Example #2

Solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary.

$$2x^2 + 9x + 7 = 3$$
$$2x^2 + 9x + 7 - 3 = 3 - 3$$
$$2x^2 + 4x + 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-9 \pm \sqrt{9^2 - 4(2)(4)}}{2(2)}$$
$$= \frac{-9 \pm \sqrt{81 - 32}}{4}$$
$$= \frac{-9 \pm \sqrt{49}}{4}$$
$$= \frac{-9 \pm 7}{4}$$

The solutions are $x = \frac{-9 + 7}{4} = \frac{-2}{4} = -\frac{1}{2}$ and $x = \frac{-9 - 7}{4} = \frac{-16}{4} = -4$.

Practice A

In Exercises 1–6, solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary.

1. $x^2 - 10x + 16 = 0$

2. $x^2 + 2x - 8 = 0$

3. $3x^2 - x - 2 = 0$

4. $x^2 + 6x = -13$

5. $-3x^2 + 5x - 1 = -7$

6. $-4x^2 + 8x + 12 = 6$

7. A square pool has a side length of x feet. A uniform border around the pool is 1 foot wide. The total area of the pool and the border is 361 square feet. What is the area of the pool?

In Exercises 8–10, determine the number of real solutions of the equation.

8. $-x^2 + 6x + 3 = 0$

9. $x^2 + 6x + 9 = 0$

10. $x^2 + 3x + 8 = 0$

In Exercises 11–13 find the number of *x*-intercepts of the graph of the function.

11. $y = -x^2 + 4x + 3$

12. $y = x^2 + 14x + 49$

13. $y = -x^2 - 8x - 18$

In Exercises 14–16, solve the equation using any method. Explain your choice of method.

14. $x^2 - 4x + 4 = 16$

15. $x^2 - 8x + 7 = 0$

16. $3x^2 + x - 5 = 0$

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Practice B

In Exercises 1–3, write the equation in standard form. Then identify the values of *a*, *b*, and *c* that you would use to solve the equation using the Quadratic Formula.

1. $x^2 + 2x = 9$ **2.** $6x - 1 = 7x^2$ **3.** $-10x + 2 = -4x^2 + 9$

In Exercises 4–11, solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary.

4. $x^2 - 8x + 16 = 0$

5. $x^2 + 10x - 11 = 0$

6. $2x^2 - 7x + 3 = 0$

7. $5x^2 + 3x - 1 = 0$

8. $5x^2 - 3x + 4 = 0$

9. $x^2 = -2x - 1$

10. $8x^2 + 9x = 3$

11. $-5x^2 + 2x = 4$

12. You launch a water balloon. The function $h = -0.08t^2 + 1.6t + 2$ models the height h (in feet) of the water balloon after t seconds.

a. After how many seconds is the water balloon at a height of 9 feet?

b. After how many seconds does the water balloon hit the ground?

In Exercises 13–15, determine the number of real solutions of the equation.

13. $4x^2 = -3x - 8$ **14.** $-2x^2 - 4x + 7 = 0$ **15.** $x^2 + 6x + 9 = 0$

In Exercises 16–18, find the number of *x*-intercepts of the graph of the function.

16. $y = 3x^2 - 6x + 3$ **17.** $y = 4x^2 + 3x + 9$ **18.** $y = -2x^2 - 3x + 1$

In Exercise 19–24, solve the equation using any method. Explain your choice of method.

19. $x^2 - 20x = 13$

20. $-7x^2 = 21x$

21. $-9x^2 = 72$

22. $7x^2 + 7 = 8 - 9x$

23. $5x^2 = 4x + 10$

24. $x^2 - 12x + 36 = 0$

25. Consider the equation $3x^2 + 5x + 6 = 0$.

a. Use the discriminant to determine the number of solutions.

b. Change the sign of c in the equation. Write the new equation.

c. Use the discriminant to determine the number of solutions of the new equation. Did your answer change? Explain.