

5.4

Proving Statements about Segments and Angles

For use with Exploration 5.4

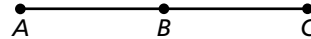
Essential Question How can you prove a mathematical statement?

A **proof** is a logical argument that uses deductive reasoning to show that a statement is true.

1 EXPLORATION: Writing Reasons in a Proof

Work with a partner. Four steps of a proof are shown. Write the reasons for each statement.

Given $AC = AB + AB$



Prove $AB = BC$

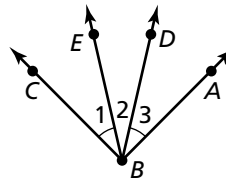
STATEMENTS	REASONS
1. $AC = AB + AB$	1. Given
2. $AB + BC = AC$	2. _____
3. $AB + AB = AB + BC$	3. _____
4. $AB = BC$	4. _____

2 EXPLORATION: Writing Steps in a Proof

Work with a partner. Six steps of a proof are shown. Complete the statements that correspond to each reason.

Given $m\angle 1 = m\angle 3$

Prove $m\angle EBA = m\angle CBD$



5.4 Proving Statements about Segments and Angles (continued)**2 EXPLORATION:** Writing Steps in a Proof (continued)

STATEMENTS	REASONS
1. _____	1. Given
2. $m\angle EBA = m\angle 2 + m\angle 3$	2. Angle Addition Postulate
3. $m\angle EBA = m\angle 2 + m\angle 1$	3. Substitution Property of Equality
4. $m\angle EBA =$ _____	4. Commutative Property of Addition
5. $m\angle 1 + m\angle 2 =$ _____	5. Angle Addition Postulate
6. _____	6. Transitive Property of Equality

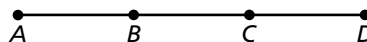
Communicate Your Answer

3. How can you prove a mathematical statement?

4. Use the given information and the figure to write a proof for the statement.

Given B is the midpoint of $\overline{AC}$.

C is the midpoint of $\overline{BD}$.



Prove $AB = CD$

5.4**Practice**

For use after Lesson 5.4

Core Concepts**Reflexive, Symmetric, and Transitive Properties of Equality**

	Real Numbers	Segment Lengths	Angle Measures
Reflexive Property	$a = a$	$AB = AB$	$m\angle A = m\angle A$
Symmetric Property	If $a = b$, then $b = a$.	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Notes:**Theorems****Properties of Segment Congruence**

Segment congruence is reflexive, symmetric, and transitive.

Reflexive	For any segment AB , $\overline{AB} \cong \overline{AB}$.
Symmetric	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Properties of Angle Congruence

Angle congruence is reflexive, symmetric, and transitive.

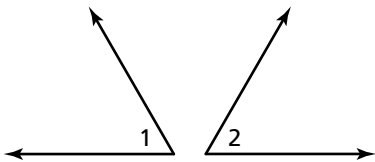
Reflexive	For any angle A , $\angle A \cong \angle A$.
Symmetric	If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
Transitive	If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Notes:

5.4 Practice (continued)

Writing a Two-Column Proof

In a proof, you make one statement at a time until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.



Proof of the Symmetric Property of Angle Congruence

Given $\angle 1 \cong \angle 2$ **Prove** $\angle 2 \cong \angle 1$

Copy or draw diagrams and label given information to help develop proofs. Do not mark or label the information in the Prove statement on the diagram.

	STATEMENTS	REASONS	
statements based on facts that you know or on conclusions from deductive reasoning	1. $\angle 1 \cong \angle 2$	1. Given	definitions, postulates, or proven theorems that allow you to state the corresponding statement
	2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles	
	3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality	
	4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles	
	The number of statements will vary.	Remember to give a reason for the last statement.	

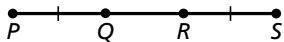
Notes:

Worked-Out Examples

Example #1

Copy and complete the proof.

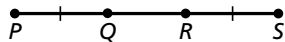
Given $PQ = RS$



Prove $PR = QS$

STATEMENTS	REASONS
1. $PQ = RS$	1. _____
2. $PQ + QR = RS + QR$	2. _____
3. _____	3. Segment Addition Postulate
4. $RS + QR = QS$	4. Segment Addition Postulate
5. $PR = QS$	5. _____

Given $PQ = RS$



Prove $PR = QS$

STATEMENTS	REASONS
1. $PQ = RS$	1. Given
2. $PQ + QR = RS + QR$	2. Addition Property of Equality
3. $PQ + QR = PR$	3. Segment Addition Postulate
4. $RS + QR = QS$	4. Segment Addition Postulate
5. $PR = QS$	5. Transitive Property of Equality

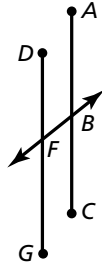
5.4 Practice (continued)

Example #2

Write a two-column proof.

Given $\overline{AB} \cong \overline{FG}$
 $\overleftrightarrow{BF}$ bisects $\overline{AC}$ and $\overline{DG}$.

Prove $\overline{BC} \cong \overline{DF}$

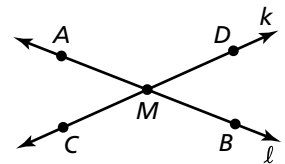


STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{FG}$	1. Given
2. $\overleftrightarrow{BF}$ bisects $\overline{AC}$ and $\overline{DG}$.	2. Given
3. $\overline{BC} \cong \overline{AB}$, $\overline{FG} \cong \overline{DF}$	3. Definition of segment bisector
4. $\overline{BC} \cong \overline{FG}$	4. Transitive Property of Equality
5. $\overline{BC} \cong \overline{DF}$	5. Transitive Property of Segment Congruence

Practice A

In Exercises 1 and 2, complete the proof.

1. **Given** $\overline{AB}$ and $\overline{CD}$ bisect each other at point M and $\overline{BM} \cong \overline{CM}$.
Prove $AB = AM + DM$

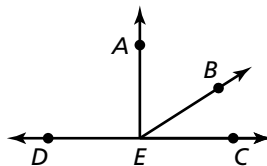


STATEMENTS	REASONS
1. $\overline{BM} \cong \overline{CM}$	1. Given
2. $\overline{CM} \cong \overline{DM}$	2. _____
3. $\overline{BM} \cong \overline{DM}$	3. _____
4. $BM = DM$	4. _____
5. _____	5. Segment Addition Postulate (Post. 1.2)
6. $AB = AM + DM$	6. _____

5.4 Practice (continued)

2. **Given** $\angle AEB$ is a complement of $\angle BEC$.

Prove $m\angle AED = 90^\circ$



STATEMENTS	REASONS
1. $\angle AEB$ is a complement of $\angle BEC$.	1. Given
2. _____	2. Definition of complementary angles
3. $m\angle AEC = m\angle AEB + m\angle BEC$	3. _____
4. $m\angle AEC = 90^\circ$	4. _____
5. $m\angle AED + m\angle AEC = 180^\circ$	5. Definition of supplementary angles
6. _____	6. Substitution Property of Equality
7. $m\angle AED = 90^\circ$	7. _____

In Exercises 3 and 4, name the property that the statement illustrates.

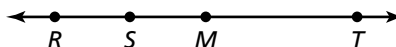
3. If $\angle RST \cong \angle TSU$ and $\angle TSU \cong \angle VWX$, then $\angle RST \cong \angle VWX$.

4. If $\overline{GH} \cong \overline{JK}$, then $\overline{JK} \cong \overline{GH}$.

5. Write a two-column proof.

Given M is the midpoint of $\overline{RT}$.

Prove $MT = RS + SM$



STATEMENTS	REASONS

Practice B

In Exercises 1 and 2, name the properties of equality that the statement illustrates.

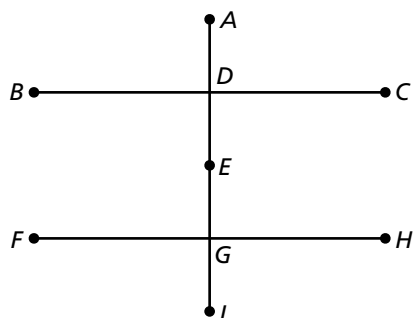
1. If $x = y$, then $2x - 6 = 2y - 6$.
2. If $m\angle A = m\angle B$ and $m\angle B = 42^\circ$, then $m\angle A + 10 = 52^\circ$.

In Exercises 3 and 4, write a two-column proof for the property.

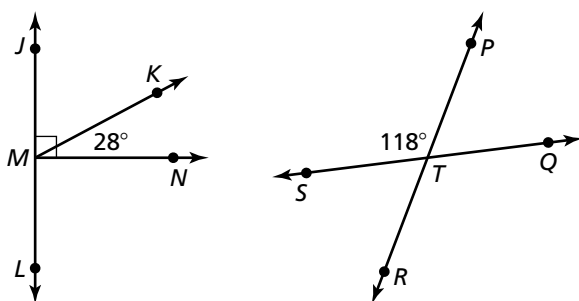
3. Symmetric Property of Segment Congruence
4. Transitive Property of Angle Congruence

In Exercises 5–7, write a two-column proof.

5. Given E bisects $\overline{AI}$, $\overline{BC}$ bisects $\overline{AE}$, and $\overline{FH}$ bisects $\overline{EI}$. Prove $\overline{AD} \cong \overline{EG}$.



6. Given $m\angle KMN = 28^\circ$ and $m\angle PTS = 118^\circ$. Prove $\angle JMK \cong \angle STR$.



7. Given $\angle ADC \cong \angle BDE$. Prove $\angle ADE \cong \angle BDC$.

