Chapter 6 | Maintaining Mathematical Proficiency

Find the measure of each angle.

1. \((5x + 2)^\circ\) \(\angle D\)
2. \((11x + 1)^\circ\) \(\angle H\)
3. \((2x + 9)^\circ\) \(\angle M\)

Write an equation of the line that passes through the given point and has the given slope.

4. \((2, 7); m = 5\)
5. \((-8, -1); m = \frac{3}{4}\)
6. \((5, -9); m = \frac{1}{6}\)

7. \((0, -8); m = \frac{3}{5}\)
8. \((-4, 3); m = \frac{1}{3}\)
9. \((2, -1); m = 5\)
6.1 Pairs of Lines and Angles

For use with Exploration 6.1

Essential Question  What does it mean when two lines are parallel, intersecting, coincident, or skew?

1 EXPLORATION: Points of Intersection

Work with a partner. Write the number of points of intersection of each pair of coplanar lines.

- a. parallel lines
- b. intersecting lines
- c. coincident lines

2 EXPLORATION: Classifying Pairs of Lines

Work with a partner. The figure shows a right rectangular prism. All its angles are right angles. Classify each of the following pairs of lines as parallel, intersecting, coincident, or skew. Justify your answers. (Two lines are skew lines when they do not intersect and are not coplanar.)

<table>
<thead>
<tr>
<th>Pair of Lines</th>
<th>Classification</th>
<th>Reason</th>
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</thead>
<tbody>
<tr>
<td>a. $\overline{AB}$ and $\overline{BC}$</td>
<td></td>
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<tr>
<td>b. $\overline{AD}$ and $\overline{BC}$</td>
<td></td>
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<tr>
<td>c. $\overline{EI}$ and $\overline{IH}$</td>
<td></td>
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<tr>
<td>d. $\overline{BF}$ and $\overline{EH}$</td>
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<tr>
<td>e. $\overline{EF}$ and $\overline{CG}$</td>
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<tr>
<td>f. $\overline{AB}$ and $\overline{GH}$</td>
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</tbody>
</table>
Work with a partner. In the figure, two parallel lines are intersected by a third line called a transversal.

a. Identify all the pairs of vertical angles. Explain your reasoning.

b. Identify all the linear pairs of angles. Explain your reasoning.

Communicate Your Answer

4. What does it mean when two lines are parallel, intersecting, coincident, or skew?

5. In Exploration 2, find three more pairs of lines that are different from those given. Classify the pairs of lines as parallel, intersecting, coincident, or skew. Justify your answers.
Core Concepts

Parallel Lines, Skew Lines, and Parallel Planes

Two lines that do not intersect are either parallel lines or skew lines. Recall that two lines are parallel lines when they do not intersect and are coplanar. Two lines are skew lines when they do not intersect and are not coplanar. Also, two planes that do not intersect are parallel planes.

Lines $m$ and $n$ are parallel lines ($m \parallel n$).

Lines $m$ and $k$ are skew lines.

Planes $T$ and $U$ are parallel planes ($T \parallel U$).

Lines $k$ and $n$ are intersecting lines, and there is a plane (not shown) containing them.

Small directed arrows, as shown on lines $m$ and $n$ above, are used to show that lines are parallel. The symbol $\parallel$ means “is parallel to,” as in $m \parallel n$.

Segments and rays are parallel when they lie in parallel lines. A line is parallel to a plane when the line is in a plane parallel to the given plane. In the diagram above, line $n$ is parallel to plane $U$.

Notes:
Parallel Postulate

If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.

There is exactly one line through \( P \) parallel to \( \ell \).

Notes:

Perpendicular Postulate

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

There is exactly one line through \( P \) perpendicular to \( \ell \).

Notes:
Perpendicular Postulate

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

There is exactly one line through \( P \) perpendicular to \( \overrightarrow{\lambda} \).

Notes:

Angles Formed by Transversals

Two angles are **corresponding angles** when they have corresponding positions. For example, \( \angle 2 \) and \( \angle 6 \) are above the lines and to the right of the transversal \( t \).

Two angles are **alternate interior angles** when they lie between the two lines and on opposite sides of the transversal \( t \).

Two angles are **alternate exterior angles** when they lie outside the two lines and on opposite sides of the transversal \( t \).

Two angles are **consecutive interior angles** when they lie between the two lines and on the same side of the transversal \( t \).

Notes:

**Worked-Out Examples**

**Example #1**

What line(s) through point \( F \) appear skew to \( \overrightarrow{EH} \)?

The only line skew to \( \overrightarrow{EH} \) that contains \( F \) is \( \overrightarrow{CF} \).

**Example #2**

Classify the pair of numbered angles.

\( \angle 1 \) and \( \angle 5 \) are corresponding angles.
Extra Practice

In Exercises 1–4, think of each segment in the diagram as part of a line. Which line(s) or plane(s) contain point B and appear to fit the description?

1. line(s) skew to \( \overrightarrow{FG} \).
2. line(s) perpendicular to \( \overrightarrow{FG} \).
3. line(s) parallel to \( \overrightarrow{FG} \).
4. plane(s) parallel to plane \( FGH \).

In Exercises 5–8, use the diagram.

5. Name a pair of parallel lines.
6. Name a pair of perpendicular lines.
7. Is \( \overrightarrow{WX} \parallel \overrightarrow{QR} \)? Explain.
8. Is \( \overrightarrow{ST} \perp \overrightarrow{NV} \)? Explain.

In Exercises 9–12, identify all pairs of angles of the given type.

9. corresponding

10. alternate interior

11. alternate exterior

12. consecutive interior
Practice B

In Exercises 1–6, use the diagram.

1. Name a pair of parallel lines.

2. Name a pair of perpendicular lines.

3. Name a pair of skew lines.

4. Name a pair of parallel planes.

5. Is line $f$ parallel to line $g$? Explain.

6. Is line $e$ perpendicular to line $g$? Explain.

In Exercises 7–11, classify the angle pair as corresponding, alternate interior, alternate exterior, or consecutive interior angles.

7. $\angle 4$ and $\angle 9$

8. $\angle 1$ and $\angle 9$

9. $\angle 1$ and $\angle 12$

10. $\angle 6$ and $\angle 11$

11. $\angle 4$ and $\angle 7$

12. Two planes are parallel and each plane contains a line. Are the two lines skew? Explain your reasoning.

13. Use the figure to decide whether the statement is true or false. Explain your reasoning.

a. The line containing the sidewalk and the line containing the center of the road are parallel to each other.

b. The line containing the center of the road is skew to the line containing the crosswalk.

c. The plane containing a stop sign is perpendicular to the plane containing the ground.