Name _____ Date _____

6.5 Using Parallel and Perpendicular Lines

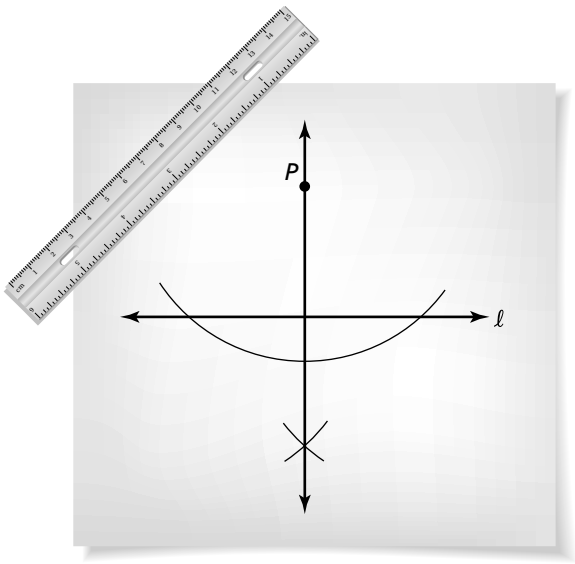
For use with Exploration 6.5

Essential Question How can you find the distance between two parallel lines?

1 EXPLORATION: Finding the Distance Between Two Parallel Lines

Work with a partner.

a. Draw a line and label it ℓ. Draw a point not on line ℓ and label it P.



b. Construct a line through point P perpendicular to line ℓ.

c. Use a centimeter ruler to measure the distance from point P to line ℓ.

Name_____ Date _____

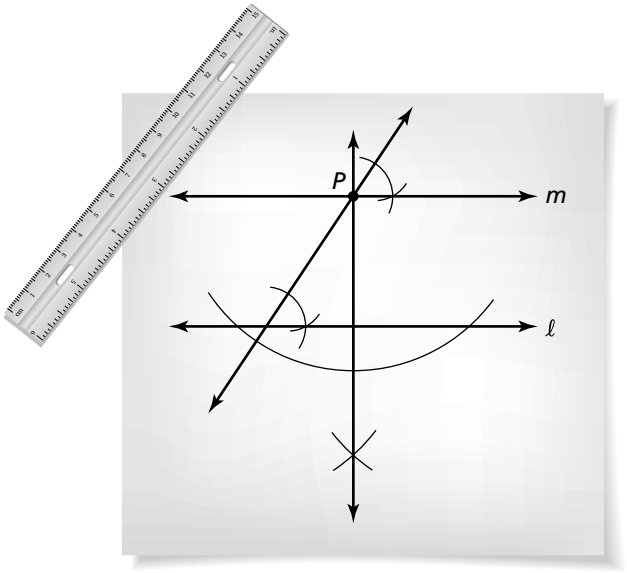
6.5 Using Parallel and Perpendicular Lines (continued)

d. Construct a line through point P parallel to line ℓ and label it m.

e. Choose any point except point P on either line and label it Q. Describe how to find the distance from point Q to the other line.

f. Find the distance from point Q to the other line. Compare this distance to the distance from point P to line ℓ.

g. Is the distance from any point on line ℓ to line m constant? Explain your reasoning.



Communicate Your Answer

2. How can you find the distance between two parallel lines?

3. Use centimeter graph paper and a centimeter ruler to find the distance between the two parallel lines.

a. $y = 2x + 2$
$y = 2x - 7$

b. $y = -x + 4$
$y = -x - 5$

Name _____ Date _____

6.5 Practice
For use after Lesson 6.5

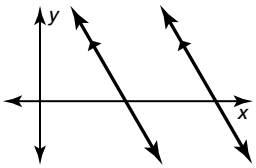
Theorems

Slopes of Parallel Lines

In a coordinate plane, two distinct nonvertical lines are parallel if and only if they have the same slope.

Any two vertical lines are parallel.

Notes:



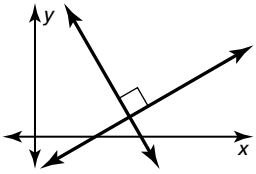
$m_1 = m_2$

Slopes of Perpendicular Lines

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Horizontal lines are perpendicular to vertical lines.

Notes:



$m_1 \bullet m_2 = -1$

Worked-Out Examples

Example #1

Find the distance from point A to the given line.

$A(-1, 4)$, line with a slope of -3 passes through $(2, -4)$

Write an equation of the line with a slope of -3 that passes through $(2, -4)$.

$y = -3x + b$

$-4 = -3(2) + b$

$-4 = -6 + b$

$+6 \quad +6$

$2 = b$

An equation is $y = -3x + 2$.

Name_____ Date _____

6.5 Practice (continued)

Use the Slopes of Perpendicular Lines Theorem to find the slope of the line perpendicular to $y = -3x + 2$.

$-3 \cdot m = -1$

$\frac{-3m}{-3} = \frac{-1}{-3}$

$m = \frac{1}{3}$

The slope of the perpendicular line is $m = \frac{1}{3}$. Find b.

$y = \frac{1}{3}x + b$

$4 = \frac{1}{3}(-1) + b$

$4 = -\frac{1}{3} + b$

$+\frac{1}{3} \quad +\frac{1}{3}$

$\frac{13}{3} = b$

The line perpendicular to $y = -3x + 2$ is $y = \frac{1}{3}x + \frac{13}{3}$. Find the point of intersection.

$y = -3x + 2$ Equation 1

$y = \frac{1}{3}x + \frac{13}{3}$ Equation 2

$-3x + 2 = \frac{1}{3}x + \frac{13}{3}$

$3 \cdot (-3x) + 3 \cdot 2 = 3 \cdot \left(\frac{1}{3}x\right) + 3 \cdot \left(\frac{13}{3}\right)$

$-9x + 6 = x + 13$

$+9x \quad +9x$

$6 = 10x + 13$

$-13 \quad -13$

$-7 = 10x$

$\frac{-7}{10} = \frac{10x}{10}$

$-\frac{7}{10} = x$

$y = -3x + 2 = -3\left(-\frac{7}{10}\right) + 2 = \frac{21}{10} + 2 = \frac{41}{10}$

So, the perpendicular lines intersect at $\left(-\frac{7}{10}, \frac{41}{10}\right)$.

6.5 Practice (continued)

Find the distance from $(-1, 4)$ to $\left(-\frac{7}{10}, \frac{41}{10}\right)$.

$$\begin{aligned}\text{distance} &= \sqrt{\left(-\frac{7}{10} - (-1)\right)^2 + \left(\frac{41}{10} - 4\right)^2} \\ &= \sqrt{\left(-\frac{7}{10} + 1\right)^2 + \left(\frac{41}{10} - 4\right)^2} \\ &= \sqrt{\left(\frac{3}{10}\right)^2 + \left(\frac{1}{10}\right)^2} \\ &= \sqrt{\frac{1}{10}} = \frac{\sqrt{10}}{10} \\ &\approx 0.3\end{aligned}$$

So, the distance from point A to the line $y = -3x + 2$ is about 0.3 unit.

Example #2

Find the distance from point A to the given line.

$A(15, -21)$, line for which $f(4) = -8$ and $f(8) = -18$

Find the slope of the line for which $f(4) = -8$ and $f(8) = -18$.

$$m = \frac{f(8) - f(4)}{8 - 4} = \frac{-18 - (-8)}{8 - 4} = \frac{-18 + 8}{8 - 4} = \frac{-10}{4}, \text{ or } -\frac{5}{2}$$

Next write an equation of the line.

$$y = -\frac{5}{2}x + b$$

$$-8 = -\frac{5}{2} \cdot (4) + b$$

$$-8 = -10 + b$$

$$\underline{+10} \quad \underline{+10}$$

$$2 = b$$

An equation is $y = -\frac{5}{2}x + 2$.

Use the Slopes of Perpendicular Lines Theorem to find the slope of the line perpendicular to $y = -\frac{5}{2}x + 2$.

$$-\frac{5}{2} \cdot m = -1$$

$$-\frac{2}{5} \cdot \left(-\frac{5}{2}m\right) = -\frac{2}{5} \cdot (-1)$$

$$m = \frac{2}{5}$$

6.5 Practice (continued)

The slope of the perpendicular line is $m = \frac{2}{5}$. Find b.

$$y = \frac{2}{5}x + b$$
$$-21 = \frac{2}{5} \cdot 15 + b$$
$$-21 = 6 + b$$
$$-27 = b$$

The line perpendicular to $5x + 2y = 4$ is $y = \frac{2}{5}x - 27$.

Find the point of intersection.

$y = -\frac{5}{2}x + 2$ Equation 1

$y = \frac{2}{5}x - 27$ Equation 2

$$-\frac{5}{2}x + 2 = \frac{2}{5}x - 27$$
$$\underline{-2} \qquad \underline{-\ 2}$$
$$-\frac{5}{2}x = \frac{2}{5}x - 29$$
$$-\frac{25}{10}x - \frac{4}{10}x = -29$$
$$-\frac{29}{10}x = -29$$
$$-\frac{10}{29} \cdot \left(-\frac{29}{10}x\right) = -29 \cdot \left(-\frac{10}{29}\right)$$
$$x = 10$$
$$y = -\frac{5}{2} \cdot 10 + 2$$
$$y = -25 + 2$$
$$y = -23$$

So, the perpendicular lines intersect at $(10, -23)$. Find the distance from $(15, -21)$ to $(10, -23)$.

$$\text{distance} = \sqrt{(10 - 15)^2 + (-23 - (-21))^2}$$
$$= \sqrt{(-5)^2 + (-2)^2}$$
$$= \sqrt{25 + 4} = \sqrt{29} \approx 5.4 \text{ units}$$

So, the distance from point A to the line $y = -\frac{5}{2}x + 2$ is about 5.4 units.

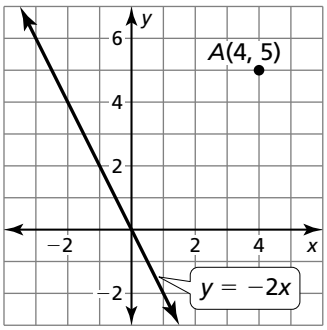
Name _____ Date _____

6.5 Practice (continued)

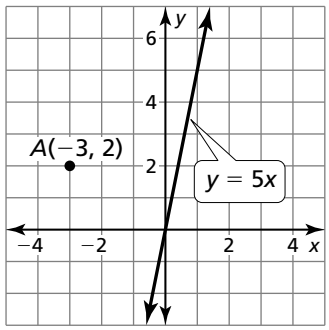
Practice A

In Exercises 1–5, find the distance from point *A* to the given line.

1.



2.



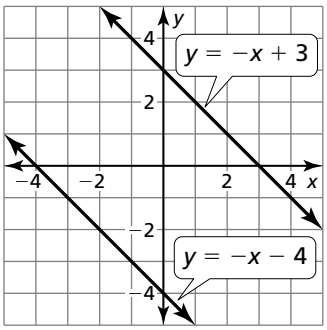
3. $A(0, 5), y = -3x - 5$

4. $A(9, 12), y = x - 3$

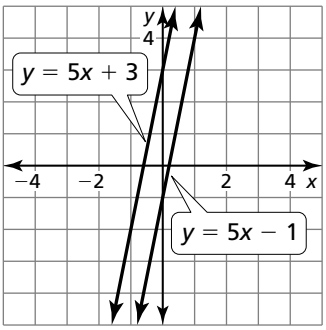
5. $A(7, -3)$, line with slope of 4 that passes through $(-2, -5)$

In Exercises 6–10, find the distance between the parallel lines.

6.



7.



8. $y = 4x - 6, y = 4x - 10$

9. $y = -7x + 3, y = -7x - 9$

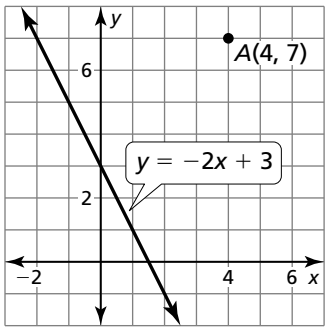
10. $y = \frac{3}{4}x - 3$, parallel line that passes through $(-2, 5)$

Name_____ Date _____

Practice B

In Exercises 1–6, find the distance from point *A* to the given line.

1.



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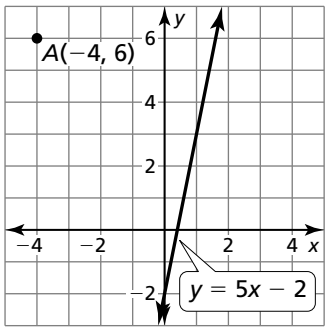


3. $A(4, 8),\ 6 = y + 2x$

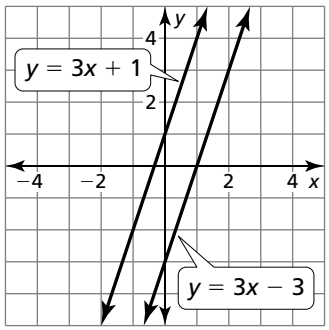
4. $A(-2, 1),\ y = \frac{1}{4}x - 3$

5. $A(-7, 3)$, line with a slope of -2 that passes through $(1, -3)$

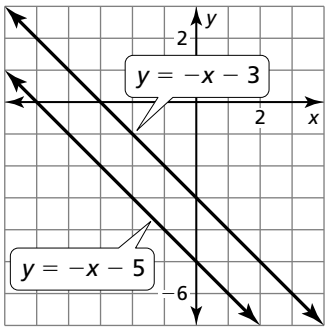
6. $A(18, 7)$, line for which $f(3) = -8$ and $f(6) = -17$

In Exercises 7–10, find the distance between the parallel lines.

7.



8.



9. $y = \frac{1}{3}x + 2$, parallel line that passes through $(0, 0)$

10. $y = -\frac{3}{2}x + 1$, parallel line that passes through $(4, -3)$

11. The diagram shows a map of a playground. The slide and the swings are connected by a straight path as shown. There is a stream that can be represented by the line $y = \frac{4}{3}x - 2$.

Each unit in the coordinate plane corresponds to 10 feet. Approximately how far is the stream from the path?