7.2 Reflections
For use with Exploration 7.2

Essential Question  How can you reflect a figure in a coordinate plane?

1 EXPLORATION: Reflecting a Triangle Using a Reflective Device

Work with a partner. Use a straightedge to draw any triangle on paper. Label it \( \triangle ABC \).

a. Use the straightedge to draw a line that does not pass through the triangle. Label it \( m \).

b. Place a reflective device on line \( m \).

c. Use the reflective device to plot the images of the vertices of \( \triangle ABC \). Label the images of vertices \( A, B, \) and \( C \) as \( A', B', \) and \( C' \), respectively.

d. Use a straightedge to draw \( \triangle A'B'C' \) by connecting the vertices.
EXPLORATION: Reflecting a Triangle in a Coordinate Plane

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any triangle and label it \( \triangle ABC \).

a. Reflect \( \triangle ABC \) in the \( y \)-axis to form \( \triangle A'B'C' \).

b. What is the relationship between the coordinates of the vertices of \( \triangle ABC \) and those of \( \triangle A'B'C' \)?

c. What do you observe about the side lengths and angle measures of the two triangles?

d. Reflect \( \triangle ABC \) in the \( x \)-axis to form \( \triangle A'B'C' \). Then repeat parts (b) and (c).

Communicate Your Answer

3. How can you reflect a figure in a coordinate plane?
Core Concepts

Reflections

A reflection is a transformation that uses a line like a mirror to reflect a figure. The mirror line is called the line of reflection.

A reflection in a line \( m \) maps every point \( P \) in the plane to a point \( P' \), so that for each point on of the following properties is true.

- If \( P \) is not on \( m \), then \( m \) is the perpendicular bisector of \( PP' \), or
- If \( P \) is on \( m \), then \( P = P' \).

Notes:

Core Concepts

Coordinate Rules for Reflections

- If \( (a, b) \) is reflected in the \( x \)-axis, then its image is the point \( (a, -b) \).
- If \( (a, b) \) is reflected in the \( y \)-axis, then its image is the point \( (-a, b) \).
- If \( (a, b) \) is reflected in the line \( y = x \), then its image is the point \( (b, a) \).
- If \( (a, b) \) is reflected in the line \( y = -x \), then its image is the point \( (-b, -a) \).

Notes:

Reflection Postulate

A reflection is a rigid motion.
Chapter 11

13. Reflections

There are two lines of symmetry.

- Glide reflection

Example #1

Graph \( \triangle JKL \) and its image after a reflection in the given line.

\( J(2, -1), K(4, -5), L(3, 1); \quad x = -1 \)

Reflect \( \triangle JKL \) in \( x = -1 \): \( J(2, -1) \rightarrow J'(-4, -1) \),
\( K(4, -5) \rightarrow K'(-6, -5) \), \( L(3, 1) \rightarrow L'(-5, 1) \)

Example #2

Graph \( \triangle RST \) with vertices \( R(4, 1), S(7, 3), \) and \( T(6, 4) \)
and its image after the glide reflection.

Translation: \((x, y) \rightarrow (x + 2, y - 1)\)
Reflection: in the line \( y = x \)

Translation \((x, y) \rightarrow (x + 2, y + 2): T(6, 4) \rightarrow T'(8, 6), S(7, 3) \rightarrow S'(9, 5), R(4, 1) \rightarrow R'(6, 3)\)

Reflection in the line \( y = x \): \( T'(8, 6) \rightarrow T''(6, 8), S'(9, 5) \rightarrow S''(5, 9), R'(6, 3) \rightarrow R''(3, 6)\)

Practice A

In Exercises 1–4, graph \( \triangle ABC \) and its image after a reflection in the given line.

1. \( A(-1, 5), B(-4, 4), C(-3, 1); \) y-axis
2. \( A(0, 2), B(4, 5), C(5, 2); \) x-axis

3. \( A(2, -1), B(-4, -2), C(-1, -3); \) y = 1
4. \( A(-2, 3), B(-2, -2), C(0, -2); \) x = -3
7.2 Practice (continued)

In Exercises 5 and 6, graph the polygon’s image after a reflection in the given line.

5. \( y = x \)  
6. \( y = -x \)

In Exercises 7 and 8, graph \( \triangle JKL \) with vertices \( J(3, 1), K(4, 2), \) and \( L(1, 3) \) and its image after the glide reflection.

7. Translation: \((x, y) \rightarrow (x - 6, y - 1)\)  
   Reflection: in the line \( y = -x \)
8. Translation: \((x, y) \rightarrow (x, y - 4)\)  
   Reflection: in the line \( x = 1 \)

In Exercises 9–12, identify the line symmetry (if any) of the word.

9. MOON  
10. WOW  
11. KID  
12. DOCK

13. You are placing a power strip along wall \( w \) that connects to two computers. Where should you place the power strip to minimize the length of the connecting cables?
Practice B

In Exercises 1 and 2, graph \( \triangle CDE \) and its image after a reflection in the given line.

1. \( C(3, 4), D(2, -1), E(0, -5); y\)-axis
2. \( C(1, 6), D(12, 2), E(7, -8); x = 8 \)

In Exercises 3 and 4, graph the polygon and its image after a reflection in the given line.

3. \( x\)-axis
4. \( y = -1 \)

In Exercises 5 and 6, graph \( \triangle ABC \) with vertices \( A(-1, 4), B(2, -1), \) and \( C(4, 3) \) and its image after the glide reflection.

5. Translation: \((x, y) \rightarrow (x + 2, y - 1)\)
   Reflection: in the line \( y = x \)
6. Translation: \((x, y) \rightarrow (x - 3, y + 1)\)
   Reflection: in the line \( y = -x \)

7. Determine the number of lines of symmetry for the figure.

8. Find point \( P \) on the \( x\)-axis so that \( AP + BP \) is a minimum.

9. Is it possible to perform two reflections of an object so that the final image is identical to the original image? If so, give an example. If not, explain your reasoning.

10. A triangle undergoes a glide reflection. Is it possible for the sides of the triangle to change length during this process? Explain your reasoning.

11. Your friend claims that it is not possible to have a glide reflection if you have one translation followed by two reflections. Is your friend correct? Explain your reasoning.