

7.4**Congruence and Transformations**

For use with Exploration 7.4

Essential Question What conjectures can you make about a figure reflected in two lines?

1 EXPLORATION: Reflections in Parallel Lines

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any scalene triangle and label it $\triangle ABC$.

- a. Draw any line \overleftrightarrow{DE} . Reflect $\triangle ABC$ in \overleftrightarrow{DE} to form $\triangle A'B'C'$.
- b. Draw a line parallel to \overleftrightarrow{DE} . Reflect $\triangle A'B'C'$ in the new line to form $\triangle A''B''C''$.
- c. Draw the line through point A that is perpendicular to \overleftrightarrow{DE} . What do you notice?
- d. Find the distance between points A and A'' . Find the distance between the two parallel lines. What do you notice?
- e. Hide $\triangle A'B'C'$. Is there a single transformation that maps $\triangle ABC$ to $\triangle A''B''C''$. Explain.
- f. Make conjectures based on your answers in parts (c)–(e). Test your conjectures by changing $\triangle ABC$ and the parallel lines.

7.4 Congruence and Transformations (continued)**2 EXPLORATION:** Reflections in Intersecting Lines

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any scalene triangle and label it $\triangle ABC$.

- a. Draw any line \overline{DE} . Reflect $\triangle ABC$ in \overline{DE} to form $\triangle A'B'C'$.
- b. Draw any line \overline{DF} so that $\angle EDF$ is less than or equal to 90° . Reflect $\triangle A'B'C'$ in \overline{DF} to form $\triangle A''B''C''$.
- c. Find the measure of $\angle EDF$. Rotate $\triangle ABC$ counterclockwise about point D twice using the measure of $\angle EDF$.
- d. Make a conjecture about a figure reflected in two intersecting lines. Test your conjecture by changing $\triangle ABC$ and the lines.

Communicate Your Answer

3. What conjectures can you make about a figure reflected in two lines?

4. Point Q is reflected in two parallel lines, \overline{GH} and \overline{JK} , to form Q' and Q'' . The distance from \overline{GH} to \overline{JK} is 3.2 inches. What is the distance QQ'' ?

7.4

Practice

For use after Lesson 7.4

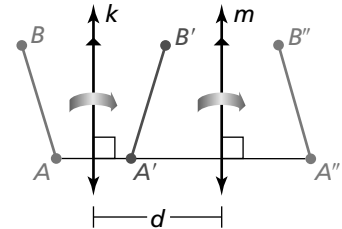
Theorems

Reflections in Parallel Lines Theorem

If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is the same as a translation.

If A'' is the image of A , then

1. $\overline{AA''}$ is perpendicular to k and m , and
2. $AA'' = 2d$, where d is the distance between k and m .

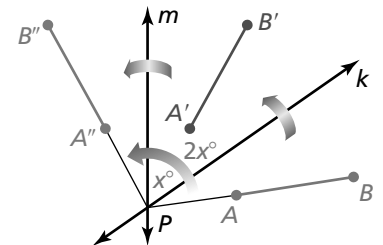


Notes:

Reflections in Intersecting Lines Theorem

If lines k and m intersect at point P , then a reflection in line k followed by a reflection in line m is the same as a rotation about point P .

The angle of rotation is $2x^\circ$, where x° is the measure of the acute or right angle formed by lines k and m .



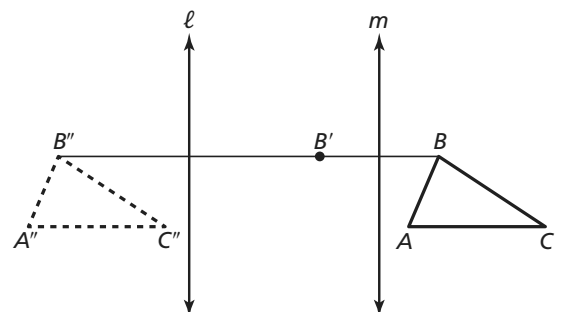
Notes:

Worked-Out Examples

Example #1

Copy the figure. Then use a compass and straightedge to construct two lines of reflection that produce a composition of reflections resulting in the same image as the given transformation.

Translation: $\triangle ABC \rightarrow \triangle A''B''C''$



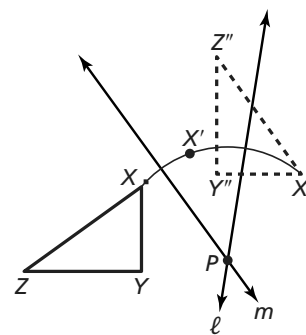
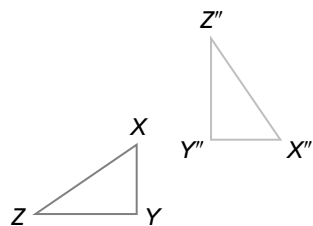
Reflect $\triangle ABC$ over line m . Then reflect over a line ℓ parallel to line m to form $\triangle A''B''C''$.

7.4 Practice (continued)

Example #2

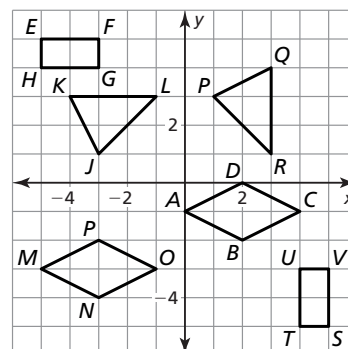
Copy the figure. Then use a compass and straightedge to construct two lines of reflection that produce a composition of reflections reflections resulting in the same image as the given transformation.

Rotation about P : $\triangle XYZ \rightarrow \triangle X''Y''Z''$

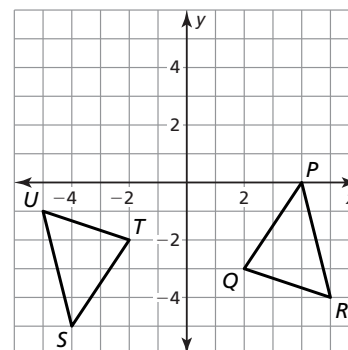


Practice A

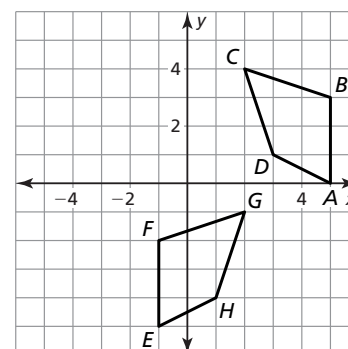
- Identify any congruent figures in the coordinate plane. Explain.



- Describe a congruence transformation that maps $\triangle PQR$ to $\triangle STU$.



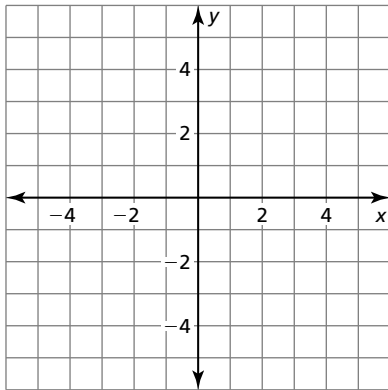
- Describe a congruence transformation that maps polygon $ABCD$ to polygon $EFGH$.



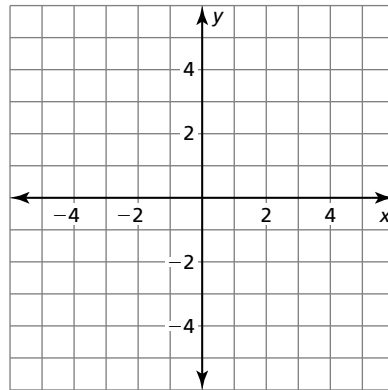
7.4 Practice (continued)

In Exercises 4 and 5, determine whether the polygons with the given vertices are congruent. Use transformations to explain your reasoning.

4. $A(2, 2)$, $B(3, 1)$, $C(1, 1)$ and
 $D(2, -2)$, $E(3, -1)$, $F(1, -1)$



5. $G(3, 3)$, $H(2, 1)$, $I(6, 2)$, $J(6, 3)$ and
 $K(2, -1)$, $L(-3, -3)$, $M(2, -2)$, $N(2, -1)$



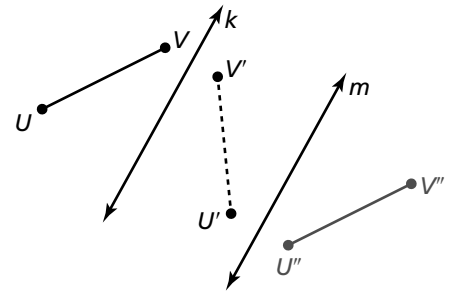
In Exercises 6–9, $k \parallel m$, \overline{UV} is reflected in line k , and $\overline{U'V'}$ is reflected in line m .

6. A translation maps \overline{UV} onto which segment?

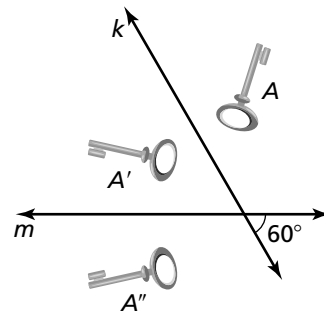
7. Which lines are perpendicular to $\overline{UU''}$?

8. Why is V'' the image of V ? Explain your reasoning.

9. If the distance between k and m is 5 inches, what is the length of $\overline{VV''}$?

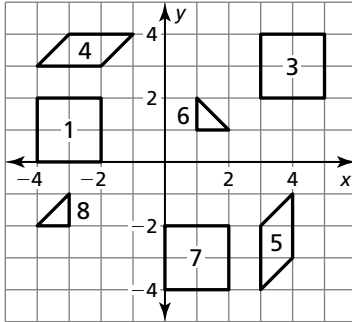


10. What is the angle of rotation that maps A onto A'' ?



Practice B

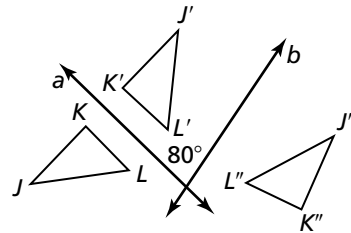
1. Identify any congruent figures in the coordinate plane. Explain.



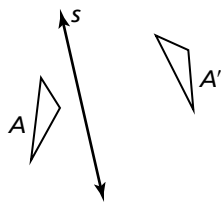
2. Determine whether the polygons with the vertices $A(0, 6)$, $B(8, 6)$, $C(6, 2)$, $D(2, 2)$ and $P(-3, -4)$, $Q(-7, -4)$, $R(-1, -8)$, $S(-5, -8)$ are congruent. Use transformations to explain your reasoning.

In Exercises 3–5, $\triangle JKL$ is reflected in line a , and $\triangle J'K'L'$ is reflected in line b .

3. \overline{JK} is perpendicular to line a and has a length of 3 units, and vertex K is 1 unit from line a . Find the distance $\overline{JJ'}$.
4. Find the angle of rotation that maps $\triangle JKL$ onto $\triangle J''K''L''$.
5. Is \overline{JK} parallel to $\overline{J''K''}$? Explain your reasoning.
6. The rotation $(x, y) \rightarrow (y, -x)$ maps P and P' . Find the measure of the acute or right angle formed by intersecting lines so that P can be mapped to P' using two reflections.
7. Is it *always*, *sometimes*, or *never* true that the composition of two reflections results in the same image as a translation? Explain your reasoning.



8. $\triangle A$ is reflected in line s to form $\triangle A'$ and then reflected in line t to form $\triangle A''$. Draw line t and intermediate $\triangle A'$ to complete the figure that represents these transformations.



9. Your friend claims that if you have a series of many parallel lines, reflecting a figure in two of the lines will produce the same result as reflecting the image in four or six of the lines. Is your friend correct? Explain your reasoning.