CHAPTER 8

Congruent Triangles

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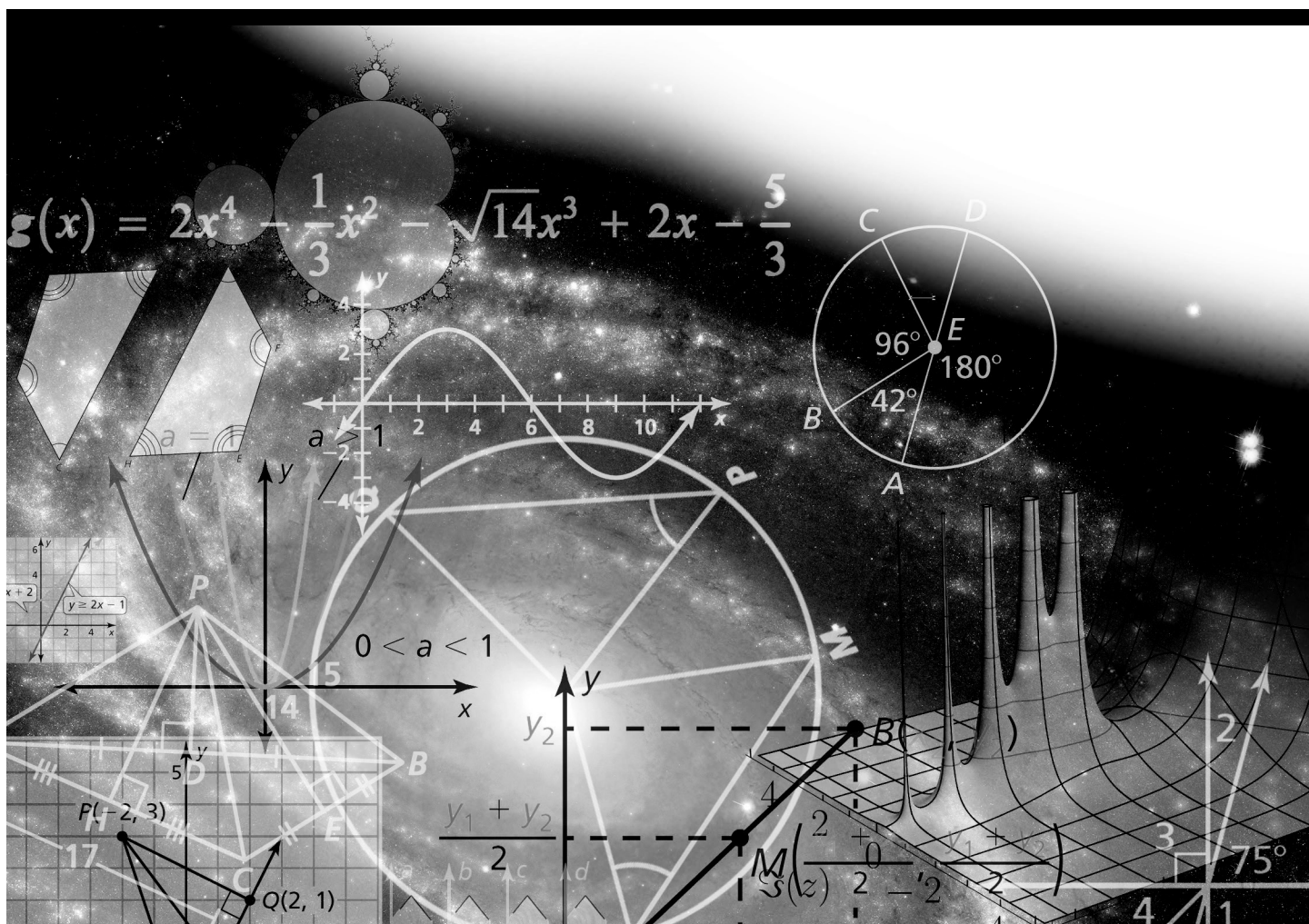
Maintaining Mathematical Proficiency

Find the coordinates of the midpoint M of the segment with the given endpoints. Then find the distance between the two points.

1. $A(3, 1)$ and $B(5, 5)$

2. $F(0, -6)$ and $G(8, -4)$

3. $P(-2, -7)$ and $B(-4, 5)$

4. $S(10, -5)$ and $T(7, -9)$

Solve the equation.

5. $9x - 6 = 7x$

6. $2r + 6 = 5r - 9$

7. $20 - 3n = 2n + 30$

8. $8t - 5 = 6t - 4$

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8.1 Angles of Triangles
For use with Exploration 8.1

Essential Question How are the angle measures of a triangle related?

1 EXPLORATION: Writing a Conjecture

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

  a. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

  b. Find the measures of the interior angles of the triangle.

  c. Find the sum of the interior angle measures.

  d. Repeat parts (a)–(c) with several other triangles. Then write a conjecture about the sum of the measures of the interior angles of a triangle.
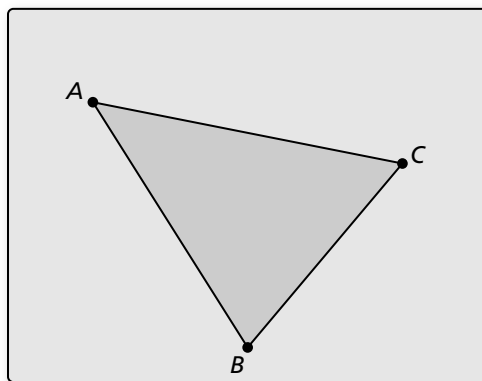
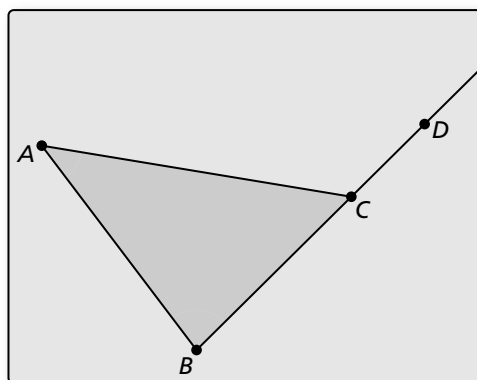**Sample**
Angles
$m\angle A = 43.67°$
$m\angle B = 81.87°$
$m\angle C = 54.46°$

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2 **EXPLORATION:** Writing a Conjecture

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

    a. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

    b. Draw an exterior angle at any vertex and find its measure.

    c. Find the measures of the two nonadjacent interior angles of the triangle.

Sample
Angles
$m\angle A = 43.67°$
$m\angle B = 81.87°$
$m\angle ACD = 125.54°$

    d. Find the sum of the measures of the two nonadjacent interior angles. Compare this sum to the measure of the exterior angle.

    e. Repeat parts (a)–(d) with several other triangles. Then write a conjecture that compares the measure of an exterior angle with the sum of the measures of the two nonadjacent interior angles.

Communicate Your Answer

    3. How are the angle measures of a triangle related?

    4. An exterior angle of a triangle measures 32°. What do you know about the measures of the interior angles? Explain your reasoning.
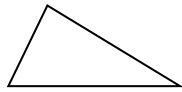
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8.1 **Practice**
For use after Lesson 8.1

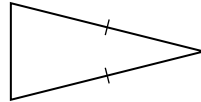## Core Concepts

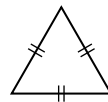### Classifying Triangles by Sides

Scalene Triangle

no congruent sides
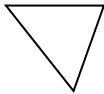
Isosceles Triangle

at least 2 congruent sides

Equilateral Triangle

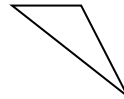

3 congruent sides

Classifying Triangles by Angles
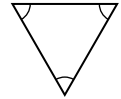
Acute Triangle

3 acute angles

Right Triangle

1 right angle

Obtuse Triangle

1 obtuse angle

Equiangular Triangle
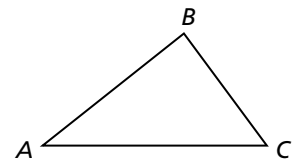

3 congruent angles

Notes:

Theorems

Triangle Sum Theorem

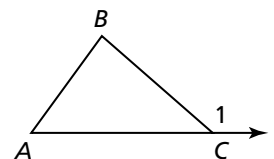The sum of the measures of the interior angles of a triangle is 180°.

Notes:

$$m\angle A + m\angle B + m\angle C = 180°$$

Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.
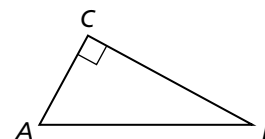
Notes:

$$m\angle 1 = m\angle A + m\angle B$$

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Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

Notes:

$$m\angle A + m\angle B = 90°$$

Worked-Out Examples

Example #1

Find the measure of the exterior angle.

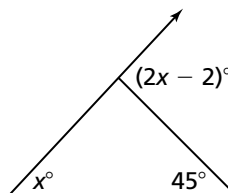$x° + 45° = (2x - 2)°$

$\quad x = 2x - 47$

$\quad -x = -47$

$\quad\quad x = 47$

$2x - 2 = 2 \cdot 47 - 2 = 94 - 2 = 92$

The exterior angle has a measure of 92°.

Example #2

Find the measure of each acute angle.

$(11x - 2)° + (6x + 7)° = 90°$

$\quad\quad 17x + 5 = 90$

$\quad\quad\quad 17x = 85$

$\quad\quad\quad\quad x = 5$
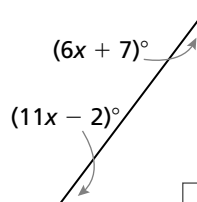
$11x - 2 = 11 \cdot 5 - 2 = 55 - 2 = 53$

$6x + 7 = 6 \cdot 5 + 7 = 30 + 7 = 37$

The two acute angles measure 37° and 53°.

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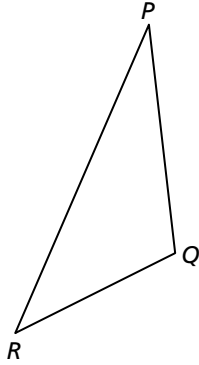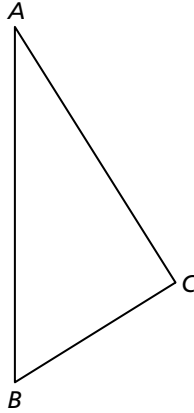
Practice A

In Exercises 1–3, classify the triangle by its sides and by measuring its angles.

1.

P

Q

R

2. *A*

C

B

3.

E

G

F

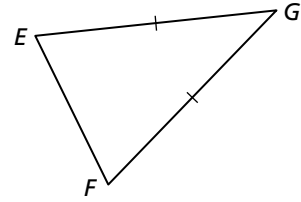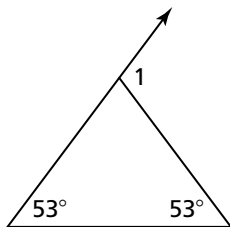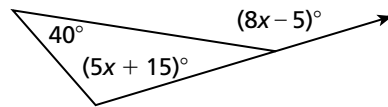**4.** Classify $\triangle ABC$ by its sides. Then determine whether it is a right triangle.

$A(6, 6), B(9, 3), C(2, 2)$

In Exercises 5 and 6, find the measure of the exterior angle.

5.

1

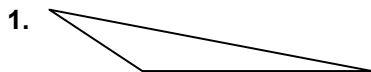53°   53°

6.

40° $(8x-5)°$

$(5x + 15)°$

7. In a right triangle, the measure of one acute angle is twice the sum of the measure of the other acute angle and 30. Find the measure of each acute angle in the right triangle.
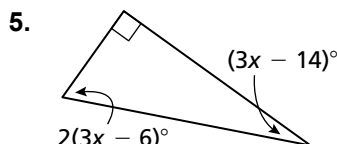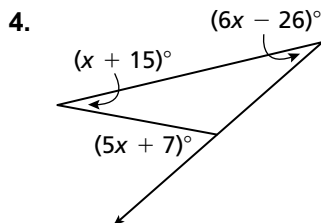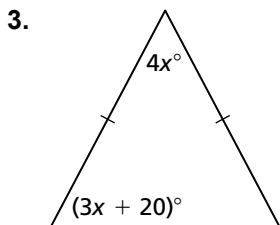
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Practice B

In Exercises 1 and 2, classify the triangle by its sides and by measuring its angles.
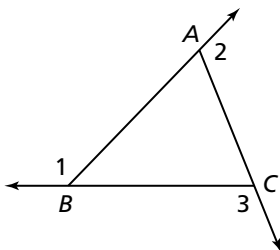
1.

2. $J(1, 2)$, $K(-4, 0)$, $L(-2, 5)$

In Exercises 3–5, find the value of *x*.

3.

$4x°$

$(3x + 20)°$

4.

$(6x - 26)°$
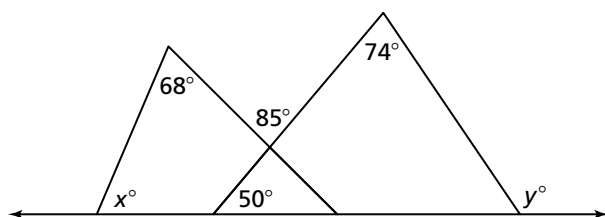
$(x + 15)°$

$(5x + 7)°$

5.

$(3x - 14)°$

$2(3x - 6)°$

6. $\triangle ABC$ is equilateral, $m\angle A = (6x + 18)°$, and $m\angle B = (3x + 2y)°$. Solve for x and y.

7. The figure shows three exterior angles of $\triangle ABC$. Show that $m\angle 1 + m\angle 2 + m\angle 3 = 360°$.

8. In the figure, solve for x and y.

9. Is it possible for a triangle to have angle measures in an extended ratio of $1 : 4 : 7$? If so, find the three angle measures. If not, explain why it is not possible.

10. Your friend says that an exterior angle can never be complementary to any of the interior angles in a triangle. Is your friend correct? Explain your reasoning.

11. In $\triangle ABC$ and $\triangle RST$, $\angle A \cong \angle R$ and $\angle B \cong \angle S$. What can you say about $\angle C$ and $\angle T$? Explain.