Name_____ Date _____

8.3 Proving Triangle Congruence by SAS

For use with Exploration 8.3

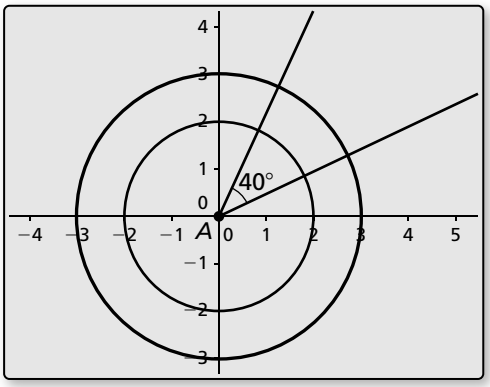
Essential Question What can you conclude about two triangles when you know that two pairs of corresponding sides and the corresponding included angles are congruent?

1 EXPLORATION: Drawing Triangles

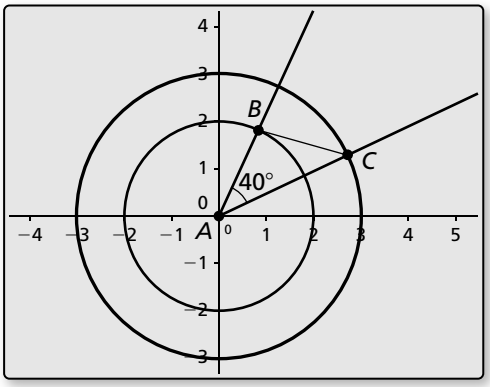
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct circles with radii of 2 units and 3 units centered at the origin. Construct a 40° angle with its vertex at the origin. Label the vertex A.



b. Locate the point where one ray of the angle intersects the smaller circle and label this point B. Locate the point where the other ray of the angle intersects the larger circle and label this point C. Then draw $\triangle ABC$.



c. Find BC, $m\angle B$, and $m\angle C$.

d. Repeat parts (a)–(c) several times, redrawing the angle in different positions. Keep track of your results by completing the table on the next page. What can you conclude?

Name _____ Date _____

8.3 Proving Triangle Congruence by SAS (continued)

1 EXPLORATION: Drawing Triangles (continued)

	A	*B*	*C*	*AB*	*AC*	*BC*	$m\angle A$	$m\angle B$	$m\angle C$
1.	(0, 0)			2	3		40°		
2.	(0, 0)			2	3		40°		
3.	(0, 0)			2	3		40°		
4.	(0, 0)			2	3		40°		
5.	(0, 0)			2	3		40°		

Communicate Your Answer

2. What can you conclude about two triangles when you know that two pairs of corresponding sides and the corresponding included angles are congruent?

3. How would you prove your conclusion in Exploration 1(d)?

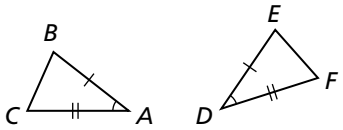
Name_____ Date _____

8.3 Practice
For use after Lesson 8.3

Theorems

Side-Angle-Side (SAS) Congruence Theorem

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.



If $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.

Notes:

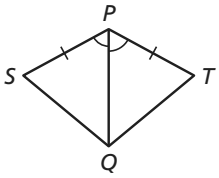
Worked-Out Examples

Example #1

Write a proof.

Given $\overline{PQ}$ bisects $\angle SPT$, $\overline{SP} \cong \overline{TP}$

Prove $\triangle SPQ \cong \triangle TPQ$



STATEMENTS	REASONS
1. $\overline{SP} \cong \overline{TP}$, $\overline{PQ}$ bisects $\angle SPT$.	**1.** Given
2. $\overline{PQ} \cong \overline{PQ}$	**2.** Reflexive Property of Congruence
3. $\angle SPQ \cong \angle TPQ$	**3.** Definition of angle bisector
4. $\triangle SPQ \cong \triangle TPQ$	**4.** SAS Congruence Theorem

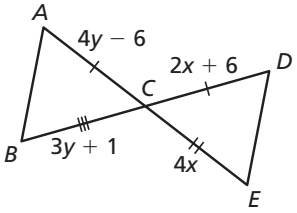
Name _____ Date _____

8.3 Practice (continued)

Example #2

Prove that △ABC ≅ △DEC. Then find the values of x and y.

Prove $\triangle ABC \cong \triangle DEC$



STATEMENTS	REASONS
1. $\overline{AC} \cong \overline{DC}$, $\overline{BC} \cong \overline{EC}$	**1.** Given (marked in diagram)
2. $\angle ACB \cong \angle DCE$	**2.** Vertical Angles Congruence Theorem
3. $\triangle ABC \cong \triangle DEC$	**3.** SAS Congruence Theorem

$$AC = CD \qquad BC = CE$$
$$4y - 6 = 2x + 6 \qquad 3y + 1 = 4x$$
$$4y = 2x + 12 \qquad 3\left(\tfrac{1}{2}x + 3\right) + 1 = 4x$$
$$y = \tfrac{1}{2}x + 3 \qquad 1.5x + 9 + 1 = 4x$$
$$1.5x + 10 = 4x$$
$$10 = 2.5x$$
$$x = 4$$

$y = \frac{1}{2} \cdot 4 + 3 = 2 + 3 = 5$

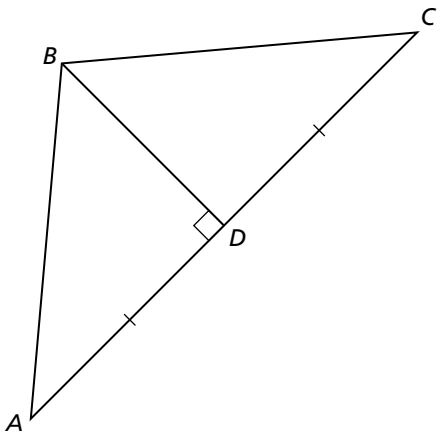
So, $x = 4$ and $y = 5$.

Practice A

In Exercises 1 and 2, write a proof.

1. Given $\overline{BD} \perp \overline{AC}$, $\overline{AD} \cong \overline{CD}$

Prove $\triangle ABD \cong \triangle CBD$



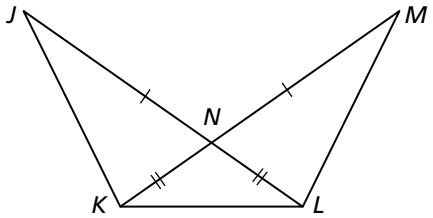
STATEMENTS	REASONS

Name_____ Date _____

8.3 Practice (continued)

2. Given $\overline{JN} \cong \overline{MN}$, $\overline{NK} \cong \overline{NL}$

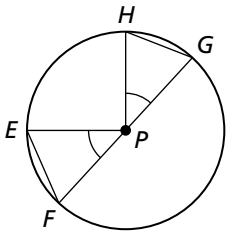
Prove $\triangle JNK \cong \triangle MNL$



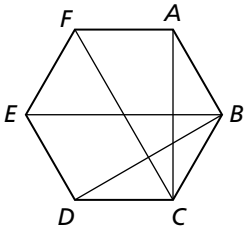
STATEMENTS	REASONS

In Exercises 3 and 4, use the given information to name two triangles that are congruent. Explain your reasoning.

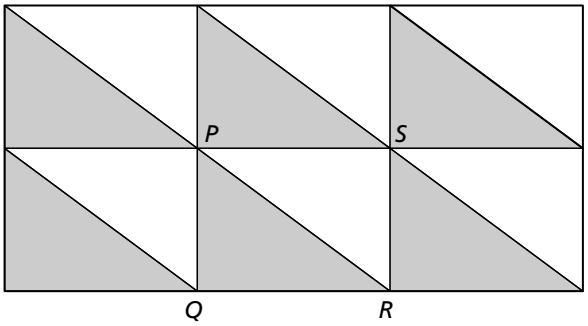
3. $\angle EPF \cong \angle GPH$, and P is the center of the circle.



4. $ABCDEF$ is a regular hexagon.



5. A quilt is made of triangles. You know $\overline{PS} \parallel \overline{QR}$ and $\overline{PS} \cong \overline{QR}$. Use the SAS Congruence Theorem to show that $\triangle PQR \cong \triangle RSP$.

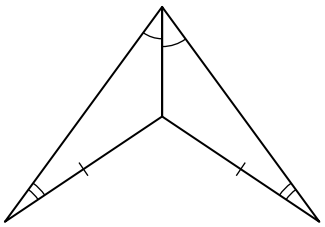


Name _____ Date _____

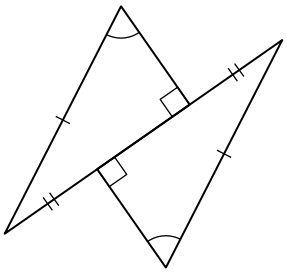
Practice B

In Exercises 1 and 2, decide whether enough information is given to prove that the triangles are congruent using the SAS Congruence Theorem. Explain.

1.

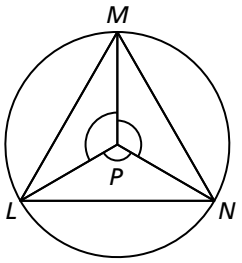


2.

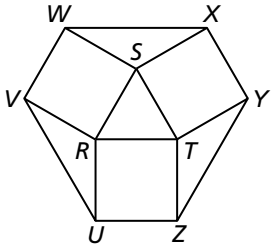


In Exercises 3 and 4, identify three congruent triangles and explain how to show that they are congruent.

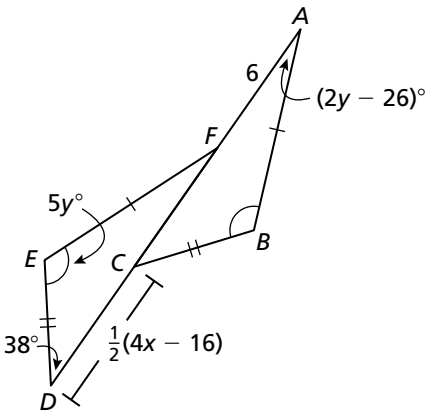
3. P is the center of the circle.



4. Three squares border equiangular and equilateral $\triangle RST$.



5. Use the information given in the figure to find the values of x and y.



6. **Given** $\overline{EB} \cong \overline{EC}$, $\triangle AED$ is equilateral and equiangular.

Prove $\triangle ACD \cong \triangle DBA$