Name ______________________________ Date __________

8.4 Equilateral and Isosceles Triangles

For use with Exploration 8.4

Essential Question What conjectures can you make about the side lengths and angle measures of an isosceles triangle?

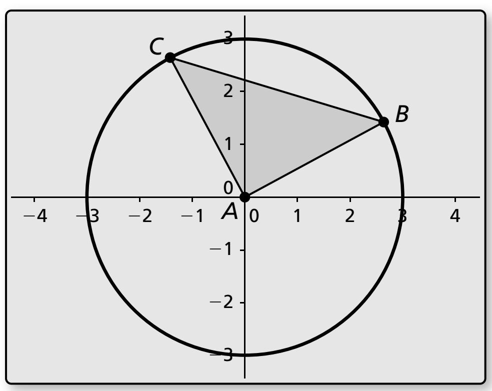
1 EXPLORATION: Writing a Conjecture about Isosceles Triangles

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct a circle with a radius of 3 units centered at the origin.

b. Construct $\triangle ABC$ so that B and C are on the circle and A is at the origin.



Sample
Points
$A(0, 0)$
$B(2.64, 1.42)$
$C(-1.42, 2.64)$
Segments
$AB = 3$
$AC = 3$
$BC = 4.24$
Angles
$m\angle A = 90°$
$m\angle B = 45°$
$m\angle C = 45°$

c. Recall that a triangle is *isosceles* if it has at least two congruent sides. Explain why $\triangle ABC$ is an isosceles triangle.

d. What do you observe about the angles of $\triangle ABC$?

e. Repeat parts (a)–(d) with several other isosceles triangles using circles of different radii. Keep track of your observations by completing the table on the next page. Then write a conjecture about the angle measures of an isosceles triangle.

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8.4 Equilateral and Isosceles Triangles (continued)

1 EXPLORATION: Writing a Conjecture about Isosceles Triangles (continued)

		A	*B*	*C*	*AB*	*AC*	*BC*	$m\angle A$	$m\angle B$	$m\angle C$
Sample	**1.**	(0, 0)	(2.64, 1.42)	(–1.42, 2.64)	3	3	4.24	90°	45°	45°
	2.	(0, 0)								
	3.	(0, 0)								
	4.	(0, 0)								
	5.	(0, 0)								

f. Write the converse of the conjecture you wrote in part (e). Is the converse true?

Communicate Your Answer

2. What conjectures can you make about the side lengths and angle measures of an isosceles triangle?

3. How would you prove your conclusion in Exploration 1(e)? in Exploration 1(f)?

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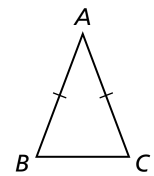
8.4 Practice
For use after Lesson 8.4

Theorems

Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

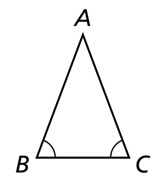
If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.



Converse of the Base Angles Theorem

If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.

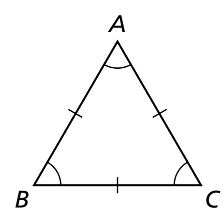


Notes:

Corollaries

Corollary to the Base Angles Theorem

If a triangle is equilateral, then it is equiangular.



Corollary to the Converse of the Base Angles Theorem

If a triangle is equiangular, then it is equilateral.

Notes:

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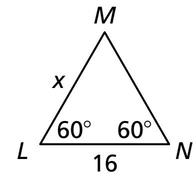
8.4 **Practice (continued)**

Worked-Out Examples

Example #1

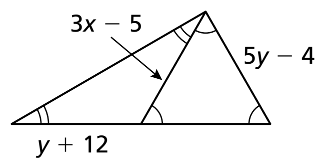
Find the value of x.



$\triangle MLN$ is an equiangular triangle and, therefore, an equilateral triangle. So, $x = 16$.

Example #2

Find the values of x and y.



By the Converse of the Base Angles Theorem:

$3x - 5 = y + 12$

$3x - 5 - 12 = y$

$y = 3x - 17$

The triangle on the right is equiangular and, therefore, equilateral.

$3x - 5 = 5y - 4$

$3x - 5 = 5(3x - 17) - 4$

$3x - 5 = 15x - 85 - 4$

$3x - 5 = 15x - 89$

$-12x = -84$

$x = 7$

$y = 3x - 17$

$y = 3 \cdot 7 - 17$

$y = 21 - 17 = 4$

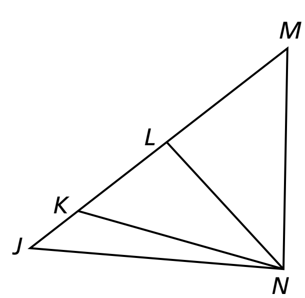
So, $x = 7$ and $y = 4$.

Practice A

In Exercises 1–4, complete the statement. State which theorem you used.

1. If $\overline{NJ} \cong \overline{NM}$, then $\angle$_______ $\cong$ $\angle$_______.

2. If $\overline{LM} \cong \overline{LN}$, then $\angle$_______ $\cong$ $\angle$_______.



3. If $\angle NKM \cong \angle NMK$, then _______ $\cong$ _______.

4. If $\angle LJN \cong \angle LNJ$, then _______ $\cong$ _______.

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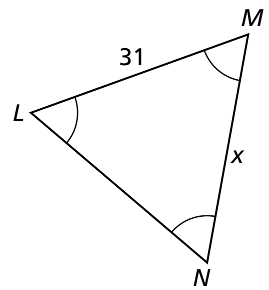
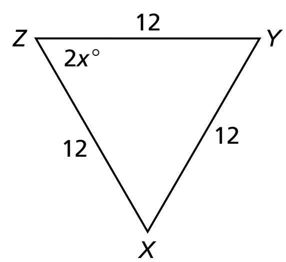
8.4 Practice (continued)

In Exercises 5 and 6, find the value of *x*.

5.

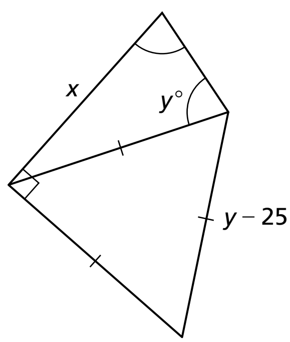


6.

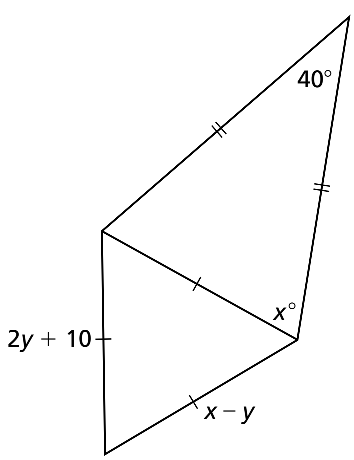


In Exercises 7 and 8, find the values of *x* and *y*.

7.



8.



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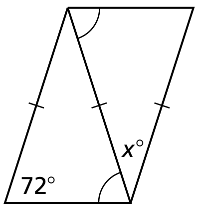
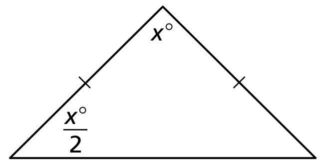
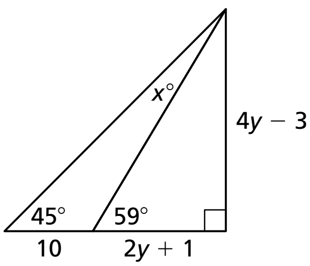
Practice B

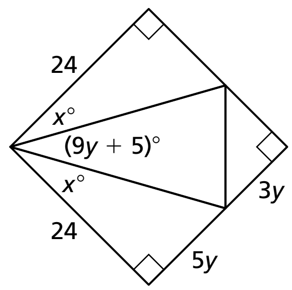
In Exercises 1 and 2, find the value of *x*.

1. 

2. 

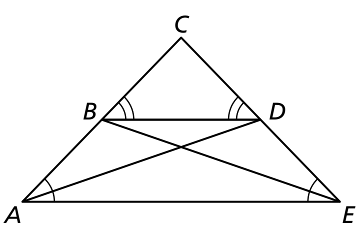
In Exercises 3 and 4, find the values of *x* and *y*.

3. 

4. 

5. **Given** $\angle CBD \cong \angle CDB$, $\angle BAE \cong \angle DEA$

   Prove $\overline{AD} \cong \overline{EB}$



6. **Given** $\angle EBC \cong \angle ECB$, $\overline{AE} \cong \overline{DE}$

   Prove $\overline{AB} \cong \overline{DC}$