Name ____ Date __________

9.2 Perpendicular and Angle Bisectors
For use with Exploration 9.2

Essential Question What conjectures can you make about a point on the perpendicular bisector of a segment and a point on the bisector of an angle?

1 EXPLORATION: Points on a Perpendicular Bisector

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

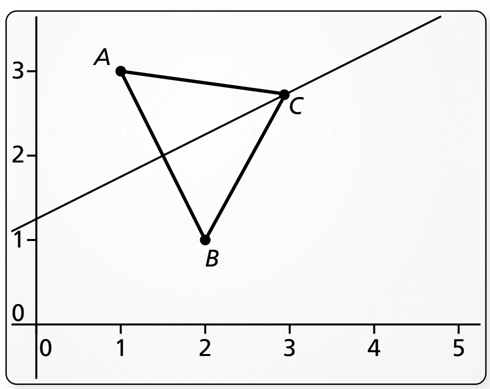
Work with a partner. Use dynamic geometry software.

a. Draw any segment and label it $\overline{AB}$. Construct the perpendicular bisector of $\overline{AB}$.

b. Label a point C that is on the perpendicular bisector of $\overline{AB}$ but is not on $\overline{AB}$.

c. Draw $\overline{CA}$ and $\overline{CB}$ and find their lengths. Then move point C to other locations on the perpendicular bisector and note the lengths of $\overline{CA}$ and $\overline{CB}$.

d. Repeat parts (a)–(c) with other segments. Describe any relationship(s) you notice.



Sample
Points
$A(1, 3)$
$B(2, 1)$
$C(2.95, 2.73)$
Segments
$AB = 2.24$
$CA = ?$
$CB = ?$
Line
$-x + 2y = 2.5$

2 EXPLORATION: Points on an Angle Bisector

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

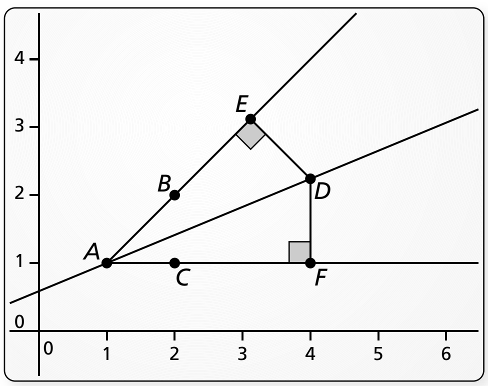
a. Draw two rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ to form $\angle BAC$. Construct the bisector of $\angle BAC$.

b. Label a point D on the bisector of $\angle BAC$.

2 EXPLORATION: Points on an Angle Bisector (continued)

c. Construct and find the lengths of the perpendicular segments from D to the sides of $\angle BAC$. Move point D along the angle bisector and note how the lengths change.

d. Repeat parts (a)–(c) with other angles. Describe any relationship(s) you notice.



Sample
Points
$A(1, 1)$
$B(2, 2)$
$C(2, 1)$
$D(4, 2.24)$
Rays
$AB = -x + y = 0$
$AC = y = 1$
Line
$-0.38x + 0.92y = 0.54$

Communicate Your Answer

3. What conjectures can you make about a point on the perpendicular bisector of a segment and a point on the bisector of an angle?

4. In Exploration 2, what is the distance from point D to $\overrightarrow{AB}$ when the distance from D to $\overrightarrow{AC}$ is 5 units? Justify your answer.

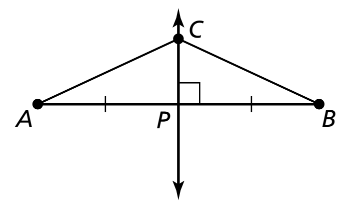
Name ______________________________ Date __________

9.2 Practice
For use after Lesson 9.2

Theorems

Perpendicular Bisector Theorem

In a plane, if a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

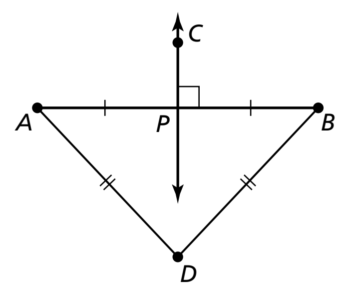


If $\overleftrightarrow{CP}$ is the $\perp$ bisector of $\overline{AB}$, then $CA = CB$.

Notes:

Converse of the Perpendicular Bisector Theorem

In a plane, if a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.

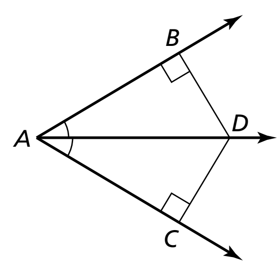


If $DA = DB$, then point D lies on the $\perp$ bisector of $\overline{AB}$.

Notes:

Angle Bisector Theorem

If a point lies on the bisector of an angle, then it is equidistant from the two sides of the angle.



If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB = DC$.

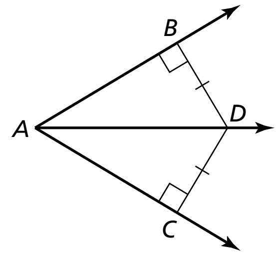
Notes:

Name____ Date__________

9.2 **Practice (continued)**

Converse of the Angle Bisector Theorem

If a point is in the interior of an angle and is equidistant from the two sides of the angle, then it lies on the bisector of the angle.



If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$,

then $\overrightarrow{AD}$ bisects $\angle BAC$.

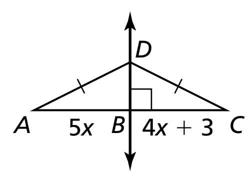
Notes:

Worked-Out Examples

Example #1

Find the indicated measure. Explain your reasoning.

AB



$AB = 15$; Because $\overleftrightarrow{DB} \perp \overleftrightarrow{AC}$ and point D is equidistant from A and C, point D is on the perpendicular bisector of $\overline{AC}$ by the Converse of the Perpendicular Bisector Theorem. By definition of segment bisector, $AB = BC$.

$AB = BC$

$5x = 4x + 3$

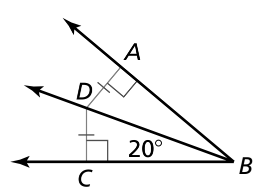
$x = 3$

$AB = 5 \cdot 3 = 15$

Example #2

Find the indicated measure. Explain your reasoning.

$m\angle ABD$



Because D is equidistant from $\overrightarrow{BC}$ and $\overrightarrow{BA}$, $\overrightarrow{BD}$ bisects $\angle ABC$ by the Converse of the Angle Bisector Theorem. So, $m\angle ABD = m\angle CBD = 20°$.

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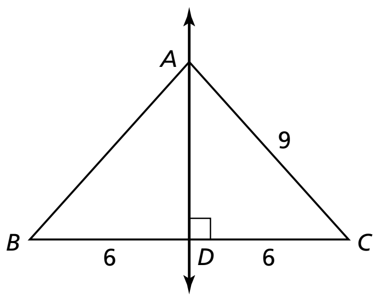
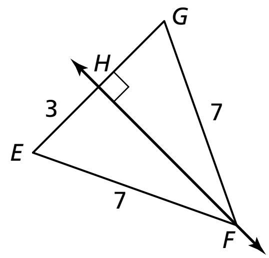
Practice A

In Exercises 1–3, find the indicated measure. Explain your reasoning.

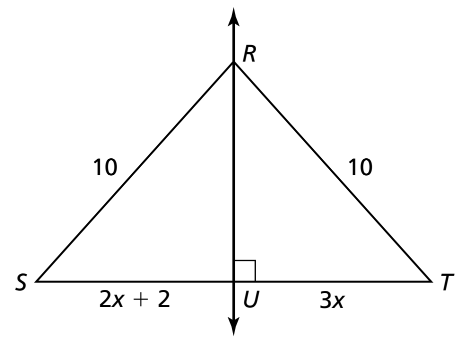
1. AB



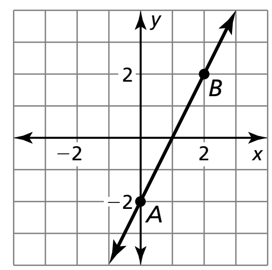
2. EG



3. SU

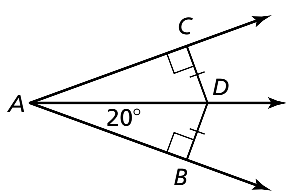


4. Find the equation of the perpendicular bisector of AB.

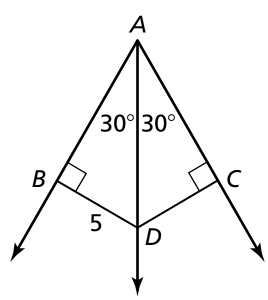


In Exercises 5–7, find the indicated measure. Explain your reasoning.

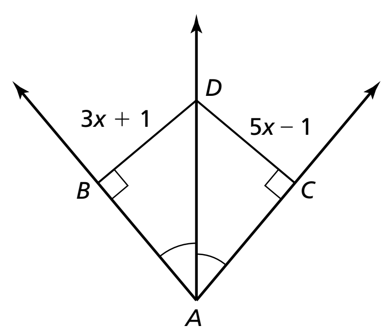
5. $m\angle CAB$



6. DC



7. BD



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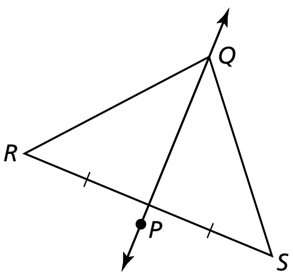
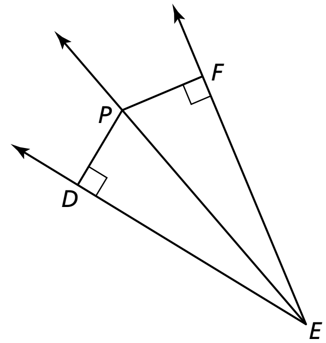
Practice B

In Exercises 1–3, tell whether the information in the diagram allows you to conclude that point *P* lies on the perpendicular bisector of $\overline{RS}$, or on the angle bisector of $\angle DEF$. Explain your reasoning.

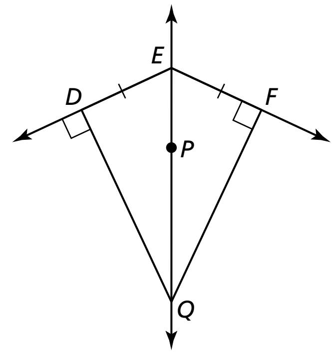
1.



2.

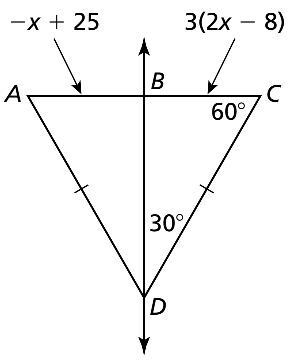


3.

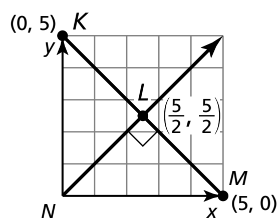


In Exercises 4–6, find the indicated measure. Explain your reasoning.

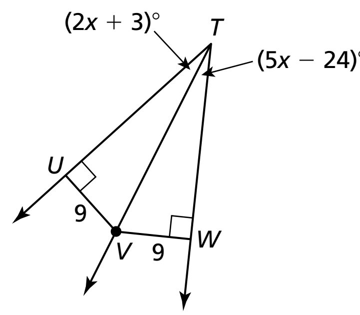
4. AC



5. $m\angle LNM$

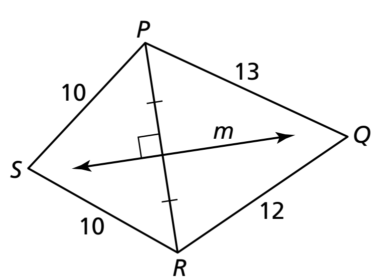


6. $m\angle UTW$



7. Write an equation of the perpendicular bisector of the segment with the endpoints $G(3, 7)$ and $H(-1, -5)$.

8. In the figure, line m is the perpendicular bisector of $\overline{PR}$. Is point Q on line m? Is point S on line m? Explain your reasoning.



9. You are installing a fountain in the triangular garden pond shown in the figure. You want to place the fountain the same distance from each side of the pond. Describe a way to determine the location of the fountain using angle bisectors.