CHAPTER 10

Similarity

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Chapter 10
Maintaining Mathematical Proficiency

Tell whether the ratios form a proportion.

1. \( \frac{3}{4}, \frac{16}{12} \)
2. \( \frac{35}{63}, \frac{45}{81} \)
3. \( \frac{12}{96}, \frac{16}{100} \)
4. \( \frac{15}{24}, \frac{75}{100} \)
5. \( \frac{17}{68}, \frac{32}{128} \)
6. \( \frac{65}{105}, \frac{156}{252} \)

Tell whether the two figures are similar. Explain your reasoning.

7. [Diagram of two similar polygons]
8. [Diagram of two similar triangles]
Essential Question  What does it mean to dilate a figure?

1 EXPLORATION: Dilating a Triangle in a Coordinate Plane

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any triangle and label it \( \triangle ABC \).

a. Dilate \( \triangle ABC \) using a scale factor of 2 and a center of dilation at the origin to form \( \triangle A'B'C' \). Compare the coordinates, side lengths, and angle measures of \( \triangle ABC \) and \( \triangle A'B'C' \).

Sample

b. Repeat part (a) using a scale factor of \( \frac{1}{2} \).

c. What do the results of parts (a) and (b) suggest about the coordinates, side lengths, and angle measures of the image of \( \triangle ABC \) after a dilation with a scale factor of \( k \)?
2 EXPLORATION: Dilating Lines in a Coordinate Plane

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw $\overline{AB}$ that passes through the origin and $\overline{AC}$ that does not pass through the origin.

a. Dilate $\overline{AB}$ using a scale factor of 3 and a center of dilation at the origin. Describe the image.

b. Dilate $\overline{AC}$ using a scale factor of 3 and a center of dilation at the origin. Describe the image.

c. Repeat parts (a) and (b) using a scale factor of $\frac{1}{4}$.

d. What do you notice about dilations of lines passing through the center of dilation and dilations of lines not passing through the center of dilation?

Communicate Your Answer

3. What does it mean to dilate a figure?

4. Repeat Exploration 1 using a center of dilation at a point other than the origin.
In your own words, write the meaning of each vocabulary term.

dilation

center of dilation

scale factor

enlargement

reduction

Core Concepts

Dilations

A dilation is a transformation in which a figure is enlarged or reduced with respect to a fixed point \(C\) called the center of dilation and a scale factor \(k\), which is the ratio of the lengths of the corresponding sides of the image and the preimage.

A dilation with center of dilation \(C\) and scale factor \(k\) maps every point \(P\) in a figure to a point \(P'\) so that the following are true.

- If \(P\) is the center point \(C\), then \(P = P'\).
- If \(P\) is not the center point \(C\), then the image point \(P'\) lies on \(CP\).
  
  The scale factor \(k\) is a positive number such that \(k = \frac{CP'}{CP}\).

- Angle measures are preserved.

Notes:
10.1 Notetaking with Vocabulary (continued)

Coordinate Rule for Dilations

If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor $k$ is the point $P'(kx, ky)$.

Notes:

Practice A

In Exercises 1–3, find the scale factor of the dilation. Then tell whether the dilation is a reduction or an enlargement.

1. [Diagram of a triangle with vertices labeled P, Q, R and a dilation point labeled C.]

2. [Diagram of another triangle with vertices labeled P, Q, R and a dilation point labeled C.]

3. [Diagram of a square with vertices labeled P, Q, R, and S and a dilation point labeled C.]

In Exercises 4 and 5, graph the polygon and its image after a dilation with scale factor $k$.

4. $A(-3, 1), B(-4, -1), C(-2, -1); k = 2$
10.1 Notetaking with Vocabulary (continued)

5. \( P(-10, 0), Q(-5, 0), R(0, 5), S(-5, 5); k = \frac{1}{5} \)

![Graph showing points and dilations]

In Exercises 6 and 7, find the coordinates of the image of the polygon after a dilation with scale factor \( k \).

6. \( A(-3, 1), B(-4, -1), C(-2, -1); k = -6 \)

7. \( P(-8, 4), Q(20, -8), R(16, 4), S(0, 12); k = -0.25 \)

8. You design a poster on an 8.5-inch by 11-inch paper for a contest at your school. The poster of the winner will be printed on a 34-inch by 44-inch canvas to be displayed. What is the scale factor of this dilation?

9. A biology book shows the image of an insect that is 10 times its actual size. The image of the insect is 8 centimeters long. What is the actual length of the insect?
Practice B

In Exercises 1 and 2, find the scale factor of the dilation. Then tell whether the dilation is a reduction or an enlargement.

1.  

2.  

In Exercises 3 and 4, copy the diagram. Then use a compass and straightedge to construct a dilation with the given center and scale factor $k$.

3.  Center $B$, $k = 2$

4.  Center $P$, $k = 75$

In Exercises 5 and 6, graph the polygon and its image after a dilation with a scale factor $k$.

5.  $J(-3, 4), K(2, 1), L(3, -2), M(-5, -4); k = 50$

6.  $V(1, 1), W(-1, 0), X(-4, 2), Y(-3, 4), Z(0, 3); k = -3$

7.  You look up at the sky at night and see the moon. It looks like it is about 2 millimeters across. If you then look at the moon through a telescope that has a magnification of 40 times, how big will it look to you through the telescope?

8.  What would it mean for an object to be dilated with a scale factor of $k = 0$?

9.  Your friend claims that if you dilate a rectangle by a certain scale factor, then the perimeter of the object also increases or decreases by the same factor. Is your friend correct? Explain your reasoning.

10. The image shows an object that has been dilated with an unknown scale factor. Use the given measures to determine the scale factor and solve for the value of $x$. 