10.4

Proving Triangle Similarity by AA For use with Exploration 10.4

Essential Question What can you conclude about two triangles when you know that two pairs of corresponding angles are congruent?



EXPLORATION: Comparing Triangles

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct $\triangle ABC$ and $\triangle DEF$ so that $m \angle A = m \angle D = 106^{\circ}, \ m \angle B = m \angle E = 31^{\circ},$ and $\triangle DEF$ is not congruent to $\triangle ABC$.



b. Find the third angle measure and the side lengths of each triangle. Record your results in column 1 of the table below.

	1.	2.	3.	4.	5.	6.
m∠A, m∠D	106°	88°	40°			
m∠B, m∠E	31°	42°	65°			
m∠C						
m∠F						
AB						
DE						
BC						
EF						
AC						
DF						

10.4 Proving Triangle Similarity by AA (continued)

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EXPLORATION: Comparing Triangles (continued)

- **c.** Are the two triangles similar? Explain.
- **d.** Repeat parts (a)–(c) to complete columns 2 and 3 of the table for the given angle measures.
- **e.** Complete each remaining column of the table using your own choice of two pairs of equal corresponding angle measures. Can you construct two triangles in this way that are *not* similar?
- **f.** Make a conjecture about any two triangles with two pairs of congruent corresponding angles.

Communicate Your Answer

- **2.** What can you conclude about two triangles when you know that two pairs of corresponding angles are congruent?
- **3.** Find *RS* in the figure at the right.



Theorems

Angle-Angle (AA) Similarity Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.



Notes:

Worked-Out Examples

Example #1

Show that the two triangles are similar.

By the Vertical Angles Congruence Theorem, $\angle MXN \cong \angle XYZ$ and you are given $\angle N \cong \angle Z$. So, by the AA Similarity Theorem, $\triangle MYN \sim \triangle XYZ$.



Example #2

Show that the two triangles are similar.

By the Reflexive Property of Congruence, $\angle Y \cong \angle Y. \ m \angle YZX = 180^{\circ} - (85 + 45)^{\circ} = 50^{\circ}$, so $\angle YZX \cong \angle W$. So, by the AA Similarity Theorem, $\triangle XYZ \sim \triangle UYW$.



10.4 Practice (continued)

Practice A

In Exercises 1 and 2, determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



In Exercises 3 and 4, show that the two triangles are similar.



10.4 Practice (continued)



In Exercises 5–13, use the diagram to complete the statement.

14. Using the diagram for Exercises 5–13, write similarity statements for each triangle similar to $\triangle EFG$.

15. Determine if it is possible for ΔHJK and ΔPQR to be similar. Explain your reasoning.

$$m \angle H = 100^\circ, m \angle K = 46^\circ, m \angle P = 44^\circ, \text{and } m \angle Q = 46^\circ$$

Practice B

In Exercises 1 and 2, determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



In Exercises 3 and 4, show that the two triangles are similar.



In Exercises 5 and 6, is it possible for $\triangle ABC$ and $\triangle XYZ$ to be similar? Explain your reasoning.

- **5.** $\angle A$ and $\angle X$ are supplementary and $\angle B$ and $\angle Z$ are complementary.
- 6. $m \angle A = 75^\circ$ and $m \angle Z = 105^\circ$
- **7.** Your friend claims that if you know three angles of one quadrilateral are congruent to three angles of another quadrilateral, then the two quadrilaterals are similar. Is your friend correct? Explain your reasoning.
- **8.** The height of the Empire State Building is 1250 feet tall. Your friend, who is 6 feet 3 inches tall, is standing nearby and casts a shadow that is 33 inches long. What is the length of the shadow of the Empire State Building?
- **9.** Use the figure to write a two-column proof.

Given $\angle ABC$ and $\angle BDC$ are right angles.

Prove $\angle A \cong \angle CBD$



10. Use the figure to write a two-column proof.

Given $\overline{YZ} \cong \overline{YV}$ $\overline{XY} \cong \overline{WY}$ **Prove** $\triangle XYW \sim \triangle VYZ$



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