

11.3

The Tangent Ratio

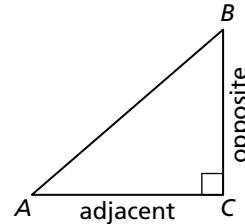
For use with Exploration 11.3

Essential Question How is a right triangle used to find the tangent of an acute angle? Is there a unique right triangle that must be used?

Let $\triangle ABC$ be a right triangle with acute $\angle A$.

The *tangent* of $\angle A$ (written as $\tan A$) is defined as follows.

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

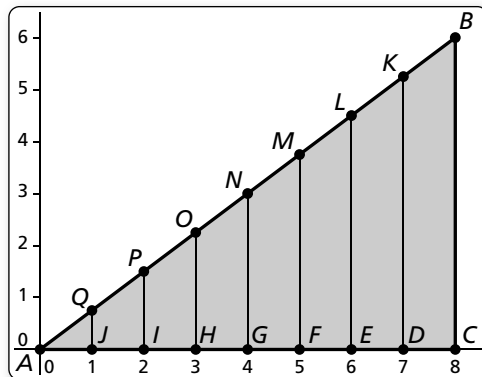


1 EXPLORATION: Calculating a Tangent Ratio

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

- a. Construct $\triangle ABC$, as shown. Construct segments perpendicular to \overline{AC} to form right triangles that share vertex A and are similar to $\triangle ABC$ with vertices, as shown.



Sample

Points

$A(0, 0)$

$B(8, 6)$

$C(8, 0)$

Angle

$m\angle BAC = 36.87^\circ$

- b. Calculate each given ratio to complete the table for the decimal value of $\tan A$ for each right triangle. What can you conclude?

Ratio	$\frac{BC}{AC}$	$\frac{KD}{AD}$	$\frac{LE}{AE}$	$\frac{MF}{AF}$	$\frac{NG}{AG}$	$\frac{OH}{AH}$	$\frac{PI}{AI}$	$\frac{QJ}{AJ}$
$\tan A$								

11.3 The Tangent Ratio (continued)**2 EXPLORATION:** Using a Calculator

Work with a partner. Use a calculator that has a tangent key to calculate the tangent of 36.87° . Do you get the same result as in Exploration 1? Explain.

Communicate Your Answer

3. Repeat Exploration 1 for $\triangle ABC$ with vertices $A(0, 0)$, $B(8, 5)$, and $C(8, 0)$.
Construct the seven perpendicular segments so that not all of them intersect \overline{AC} at integer values of x . Discuss your results.

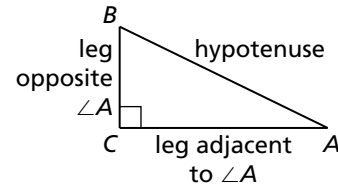
4. How is a right triangle used to find the tangent of an acute angle? Is there a unique right triangle that must be used?

11.3**Practice**

For use after Lesson 11.3

Core Concepts**Tangent Ratio**Let $\triangle ABC$ be a right triangle with acute $\angle A$.The tangent of $\angle A$ (written as $\tan A$) is defined as follows.

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

**Notes:****Worked-Out Examples****Example #1**

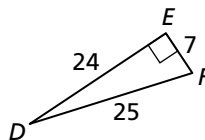
Find the tangents of the acute angles in the right triangle. Write each answer as a fraction and as a decimal rounded to four decimal places.

$$\tan D = \frac{\text{opp}}{\text{adj}}$$

$$\tan D = \frac{7}{24} \approx 0.2917$$

$$\tan F = \frac{\text{opp}}{\text{adj}}$$

$$\tan F = \frac{24}{7} \approx 3.4286$$

**Example #2**Find the value of x . Round your answer to the nearest tenth.

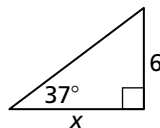
$$\tan 37^\circ = \frac{\text{opp}}{\text{adj}}$$

$$\tan 37^\circ = \frac{6}{x}$$

$$x \cdot \tan 37^\circ = 6$$

$$\frac{x \cdot \tan 37^\circ}{\tan 37^\circ} = \frac{6}{\tan 37^\circ}$$

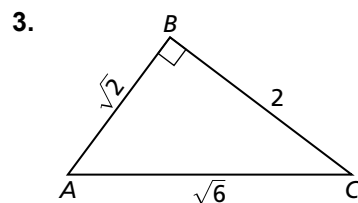
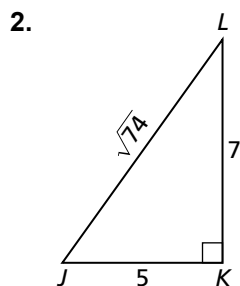
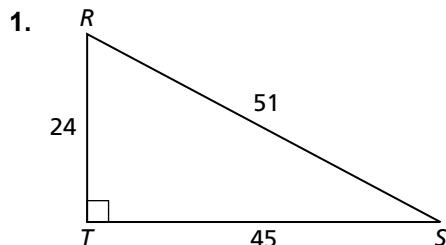
$$x = \frac{6}{\tan 37^\circ} \approx 8.0$$



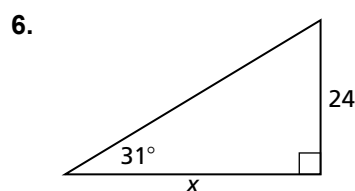
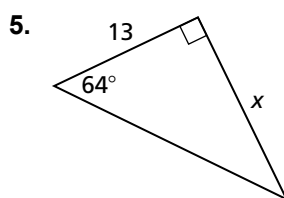
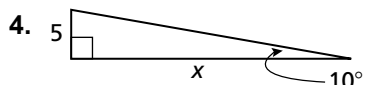
11.3 Practice (continued)

Practice A

In Exercises 1–3, find the tangents of the acute angles in the right triangle. Write each answer as a fraction and as a decimal rounded to four decimal places.



In Exercises 4–6, find the value of x . Round your answer to the nearest tenth.

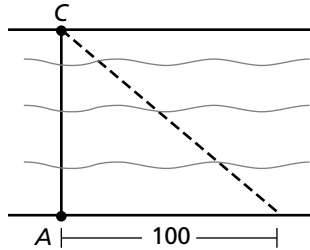


7. In $\triangle CDE$, $\angle E = 90^\circ$ and $\tan C = \frac{4}{3}$. Find $\tan D$? Write your answer as a fraction.

11.3 Practice (continued)

8. An environmentalist wants to measure the width of a river to monitor its erosion. From point A , she walks downstream 100 feet and measures the angle from this point to point C to be 40° .

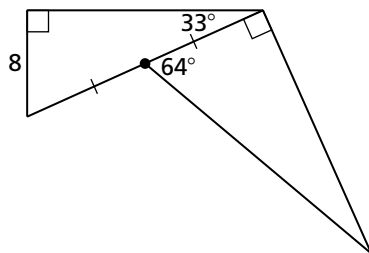
- a. How wide is the river? Round to the nearest tenth.



- b. One year later, the environmentalist returns to measure the same river. From point A , she again walks downstream 100 feet and measures the angle from this point to point C to be now 51° . By how many feet has the width of the river increased?

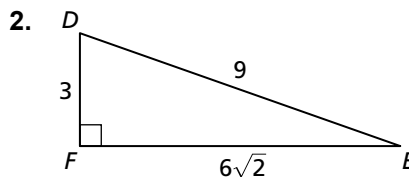
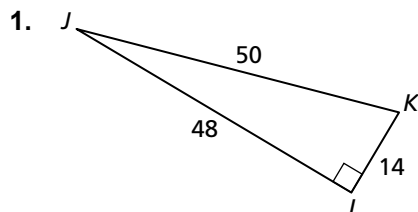
9. A boy flies a kite at an angle of elevation of 18° . The kite reaches its maximum height 300 feet away from the boy. What is the maximum height of the kite? Round to the nearest tenth.

10. Find the perimeter of the figure.



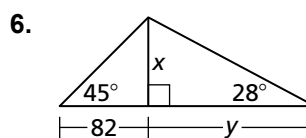
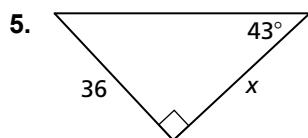
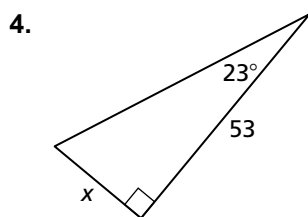
Practice B

In Exercises 1 and 2, find the tangents of the acute angles in the right triangle. Write each answer as a fraction and as a decimal rounded to four decimal places.

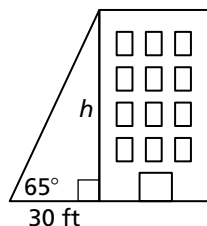


3. Draw and label the sides and angles of a triangle for which the tangents of the acute angles are equal to 1.

In Exercises 4–6, find the value(s) of the variable(s). Round your answer(s) to the nearest tenth.



7. A surveyor is standing 30 feet from the base of a tall building. The surveyor measures the angle of elevation from the ground to the top of the building to be 65° . Find the height h of the building to the nearest foot.



8. In the diagram, $\overline{RQ} \perp \overline{PQ}$, $m\angle QPS = 32^\circ$, $m\angle RPS = 24^\circ$, and $PQ = 14$. Find RS to the nearest tenth of a unit.

