## CHAPTER 1

## Absolute Value and Piecewise Functions

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## Chapter Maintaining Mathematical Proficiency

Let $f(x)=2 x$. Graph $f$ and $g$. Describe the transformation from the graph of $f$ to the graph of $g$.

1. $g(x)=f(x)-4$
2. $g(x)=f(x+2)$
3. $g(x)=f\left(\frac{1}{2} x\right)$
4. $g(x)=3 f(x)$
5. Describe the transformation from the graph of $f(x)=x$ to the graph of $h(x)=-\frac{1}{3} x+2$.

Graph the figure and its image after a reflection in the line $\boldsymbol{y}=\boldsymbol{x}$.
6. $\overline{L M}$ with endpoints $L(2,-4)$ and $M(2,0)$
7. $\overline{S T}$ with endpoints $S(-2,5)$ and $T(-4,-1)$
8. $\triangle A B C$ with vertices $A(6,4), B(6,-1)$, and $C(-2,0)$
9. $\square E F G H$ with vertices $E(-2,-4), F(4,-4), G(4,3)$, and $H(-2,3)$
10. After a reflection in the line $y=-x$, a point originally in Quadrant I will be in which Quadrant?
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## Absolute Value Functions

Essential Question How do the values of $a, h$, and $k$ affect the graph of the absolute value function $g(x)=a|x-h|+k$ ?

## 1 EXPLORATION: Identifying Graphs of Absolute Value Functions

Work with a partner. Match each absolute value function with its graph. Then use a graphing calculator to verify your answers.
a. $g(x)=-|x-2|$
b. $\quad g(x)=|x-2|+2$
c. $g(x)=-|x+2|-2$
d. $g(x)=|x-2|-2$
e. $g(x)=2|x-2|$
f. $g(x)=-|x+2|+2$
A.

B.

C.

D.

E.

F.

$\qquad$

### 1.1 Absolute Value Functions (continued)

## Communicate Your Answer

2. How do the values of $a, h$, and $k$ affect the graph of the absolute value function $g(x)=a|x-h|+k$ ?
3. Write the equation of the absolute value function whose graph is shown. Use a graphing calculator to verify your equation.

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## Notes:

## Core Concepts

## Absolute Value Function

An absolute value function is a function that contains an absolute value expression. The parent absolute value function is $f(x)=|x|$. The graph of $f(x)=|x|$ is V -shaped and symmetric about the $y$-axis. The vertex is the point where the graph changes direction. The vertex of the graph of $f(x)=|x|$ is $(0,0)$.


The domain of $f(x)=|x|$ is all real numbers.
The range is $y \geq 0$.

## Notes:

## Vertex Form of an Absolute Value Function

An absolute value function written in the form $g(x)=a|x-h|+k$, where $a \neq 0$, is in vertex form. The vertex of the graph of $g$ is $(h, k)$.

Any absolute value function can be written in vertex form, and its graph is symmetric about the line $x=h$.

Notes:
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### 1.1 Practice (continued)

## Worked-Out Examples

## Example \#1

Graph the function. Compare the graph to the graph of $f(x)=|x-6|$.
$h(x)=|x-6|+2$

| $\boldsymbol{x}$ | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{h}(\boldsymbol{x})$ | 4 | 3 | 2 | 3 | 4 |



The function $h$ is of the form $y=f(x)+k$, where $k=2$. So, the graph of $h$ is a vertical translation 2 units up of the graph of $f(x)=|x-6|$.

## Example \#2

Graph the function. Compare the graph to the graph of $f(x)=|x-6|$.
$n(x)=\frac{1}{2}|x-6|$

| $\boldsymbol{x}$ | 2 | 4 | 6 | 8 | 10 |
| :--- | :--- | :--- | :--- | :--- | :---: |
| $\boldsymbol{n}(\boldsymbol{x})$ | 2 | 1 | 0 | 1 | 2 |



The function $n$ is of the form $y=a f(x)$, where $a=\frac{1}{2}$. So, the graph of $n$ is a vertical shrink of the graph of $f(x)=|x-6|$ by a factor of $\frac{1}{2}$.
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### 1.1 Practice (continued)

## Practice A

In Exercises 1-4, graph the function. Compare the graph to the graph of $f(x)=|x|$. Describe the domain and range.

1. $t(x)=\frac{1}{2}|x|$

| $\boldsymbol{x}$ | -4 | -2 | 0 | 2 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{t}(\mathbf{x})$ |  |  |  |  |  |


3. $p(x)=|x|-3$

| $\boldsymbol{x}$ | -2 | -1 | 0 | 1 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{p}(\boldsymbol{x})$ |  |  |  |  |  |


2. $u(x)=-|x|$

| $\boldsymbol{x}$ | -2 | -1 | 0 | 1 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{u}(\boldsymbol{x})$ |  |  |  |  |  |


4. $r(x)=|x+2|$

| $\boldsymbol{x}$ | -4 | -3 | -2 | -1 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{r}(\boldsymbol{x})$ |  |  |  |  |  |


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## Practice B

In Exercises 1-4, graph the function. Compare the graph to the graph of $f(x)=|x|$. Describe the domain and range.

1. $m(x)=|x-3|$
2. $t(x)=4|x|$
3. $g(x)=-3|x|$
4. $z(x)=-\frac{4}{3}|x|$

In Exercises 5 and 6, graph the function. Compare the graph to the graph of $f(x)=|x-2|+4$.
5. $k(x)=|x-5|+4$
6. $q(x)=|x-2|-3$

In Exercises 7 and 8, compare the graphs. Find the value of $\boldsymbol{h}, \boldsymbol{k}$, or $\boldsymbol{a}$.
7.

8.


In Exercises 9 and 10, write an equation that represents the given transformation(s) of the graph of $g(x)=|x|$.
9. horizontal translation 7 units right
10. vertical shrink by a factor of $\frac{1}{3}$ and a reflection in the $x$-axis

In Exercises 11 and 12, graph and compare the two functions.
11. $c(x)=|x-4|+3 ; d(x)=|6 x-4|+3$
12. $p(x)=|x+1|-2 ; q(x)=\left|-\frac{2}{5} x+1\right|-2$
13. Graph $y=-\frac{3}{2}|x+3|-5$ and $y=-8$ in the same coordinate plane.

Use the graph to solve the equation $-\frac{3}{2}|x+3|-5=-8$. Check your solutions.

