

1.4

Piecewise Functions

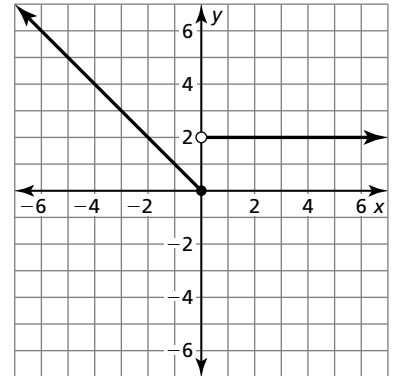
For use with Exploration 1.4

Essential Question How can you describe a function that is represented by more than one equation?

1 EXPLORATION: Writing Equations for a Function

Work with a partner.

a. Does the graph represent y as a function of x ? Justify your conclusion.



b. What is the value of the function when $x = 0$? How can you tell?

c. Write an equation that represents the values of the function when $x \leq 0$.

$$f(x) = \underline{\hspace{2cm}}, \text{ if } x \leq 0$$

d. Write an equation that represents the values of the function when $x > 0$.

$$f(x) = \underline{\hspace{2cm}}, \text{ if } x > 0$$

e. Combine the results of parts (c) and (d) to write a single description of the function.

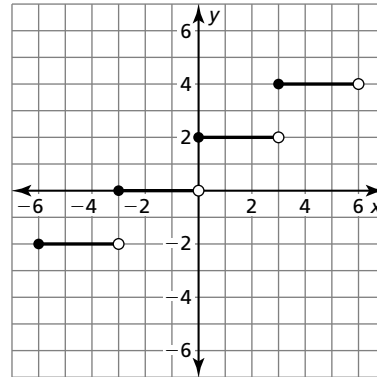
$$f(x) = \begin{cases} \underline{\hspace{2cm}}, & \text{if } x \leq 0 \\ \underline{\hspace{2cm}}, & \text{if } x > 0 \end{cases}$$

1.4 Piecewise Functions (continued)

2 EXPLORATION: Writing Equations for a Function

Work with a partner.

- a. Does the graph represent y as a function of x ? Justify your conclusion.

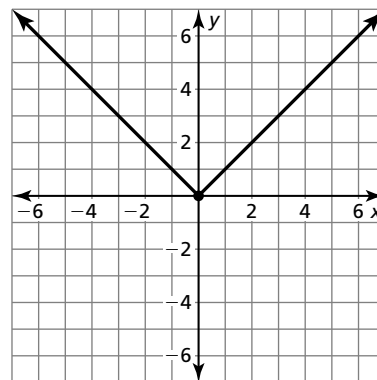


- b. Describe the values of the function for the following intervals.

$$f(x) = \begin{cases} \underline{\hspace{2cm}}, & \text{if } -6 \leq x < -3 \\ \underline{\hspace{2cm}}, & \text{if } -3 \leq x < 0 \\ \underline{\hspace{2cm}}, & \text{if } 0 \leq x < 3 \\ \underline{\hspace{2cm}}, & \text{if } 3 \leq x < 6 \end{cases}$$

Communicate Your Answer

3. How can you describe a function that is represented by more than one equation?



4. Use two equations to describe the function represented by the graph.

1.4

Practice

For use after Lesson 1.4

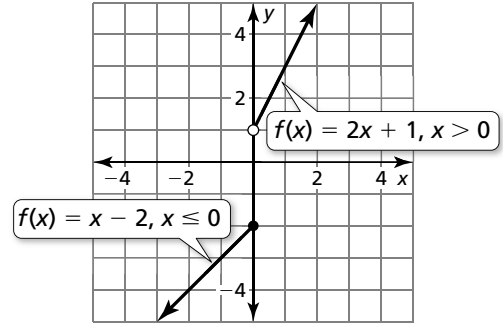
Core Concepts

Piecewise Function

A **piecewise function** is a function defined by two or more equations. Each “piece” of the function applies to a different part of its domain. An example is shown below.

$$f(x) = \begin{cases} x - 2, & \text{if } x \leq 0 \\ 2x + 1, & \text{if } x > 0 \end{cases}$$

- The expression $x - 2$ represents the value of f when x is less than or equal to 0.
- The expression $2x + 1$ represents the value of f when x is greater than 0.



Notes:

Worked-Out Examples

Example #1

Evaluate the function.

$$g(x) = \begin{cases} -x + 4, & \text{if } x \leq -1 \\ 3, & \text{if } -1 < x < 2 \\ 2x - 5, & \text{if } x \geq 2 \end{cases}; g(2)$$

$$g(x) = 2x - 5$$

$$g(2) = 2(2) - 5$$

$$g(2) = 4 - 5$$

$$g(2) = -1$$

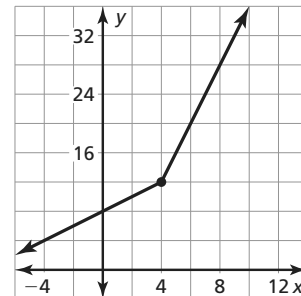
The value of g is -1 when $x = 2$.

Example #2

Graph the function. Describe the domain and range.

$$y = \begin{cases} x + 8, & \text{if } x < 4 \\ 4x - 4, & \text{if } x \geq 4 \end{cases}$$

x	0	2	4	5	6
y	8	10	12	16	20



The domain is all real numbers.

The range is all real numbers.

1.4 Practice (continued)

Practice A

In Exercise 1–9, evaluate the function.

$$f(x) = \begin{cases} 3x - 1, & \text{if } x \leq 1 \\ 1 - 2x, & \text{if } x > 1 \end{cases}$$

$$g(x) = \begin{cases} 3x - 1, & \text{if } x \leq -3 \\ 2, & \text{if } -3 < x < 1 \\ -3x, & \text{if } x \geq 1 \end{cases}$$

1. $f(0)$ 2. $f(1)$ 3. $f(5)$

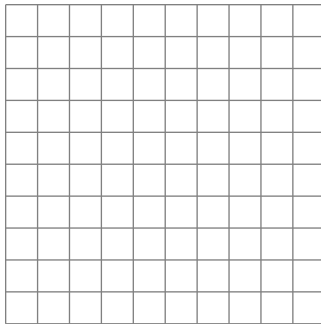
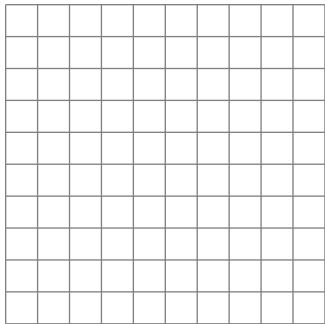
4. $f(-4)$ 5. $g(0)$ 6. $g(-3)$

7. $g(1)$ 8. $g(3)$ 9. $g(-5)$

In Exercise 10–13, graph the function. Describe the domain and range.

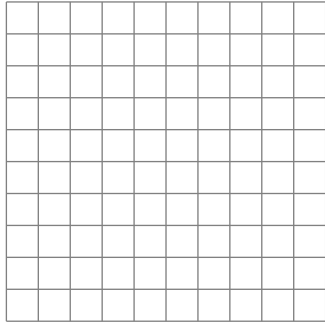
10. $y = \begin{cases} -4x, & \text{if } x \leq 0 \\ 4, & \text{if } x > 0 \end{cases}$

11. $y = \begin{cases} 4 - x, & \text{if } x < 2 \\ x + 3, & \text{if } x \geq 2 \end{cases}$

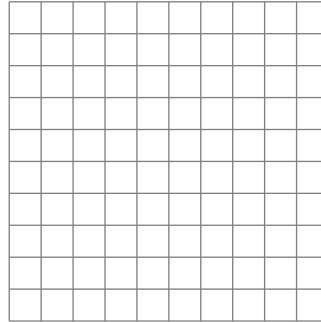


1.4 Practice (continued)

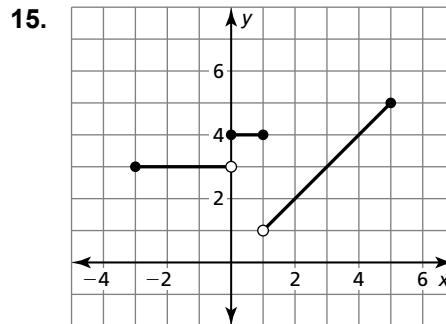
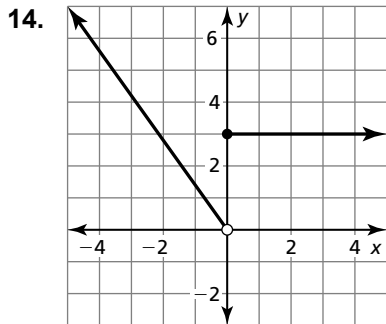
12.
$$y = \begin{cases} 2x, & \text{if } x < -2 \\ 2, & \text{if } -2 \leq x < 2 \\ -2x, & \text{if } x \geq 2 \end{cases}$$



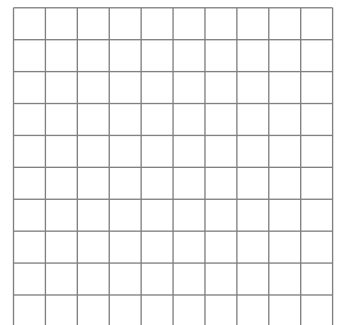
13.
$$y = \begin{cases} -1, & \text{if } x \leq -1 \\ 0, & \text{if } -1 < x < 2 \\ 1, & \text{if } x \geq 2 \end{cases}$$



In Exercise 14 and 15, write a piecewise function for the graph.



16. A postal service charges \$4 for shipping any package weighing up to but not including 1 pound and \$1 for each additional pound or portion of a pound up to but not including 5 pounds. Packages 5 pounds or over have different rates. Write and graph a step function that shows the relationship between the number x of pounds a package weighs and the total cost y for postage.



Practice B

In Exercises 1–6, evaluate the function.

$$f(x) = \begin{cases} -x + 2, & \text{if } x < -3 \\ 7, & \text{if } -3 \leq x < 0 \\ 3x - 1, & \text{if } x \geq 0 \end{cases}$$

1. $f(-5)$ 2. $f(4)$ 3. $f(1)$
 4. $f(0)$ 5. $f\left(-\frac{1}{2}\right)$ 6. $f(-3)$

7. The total cost (in dollars) of ordering graduation announcements is represented by the piecewise function

$$c(x) = \begin{cases} 1.5x + 15, & \text{if } 0 \leq x < 25 \\ 1.25x + 15, & \text{if } 25 \leq x < 40 \\ x + 15, & \text{if } x \geq 40 \end{cases}$$

- a. Determine the cost of ordering 25 announcements. Then determine the cost of ordering 24 announcements.
 b. For what number of announcements less than 25 is it financially better to purchase 25 announcements?
 c. For what number of announcements less than 40 is it financially better to purchase 40 announcements?

In Exercises 8–11, graph the function. Describe the domain and range.

8. $f(x) = \begin{cases} -x + 5, & \text{if } x < 5 \\ x - 5, & \text{if } x \geq 5 \end{cases}$ 9. $f(x) = \begin{cases} 2x - 3, & \text{if } x \leq -1 \\ 2x + 2, & \text{if } x > -1 \end{cases}$
 10. $f(x) = \begin{cases} -x + 1, & \text{if } x < -3 \\ 4, & \text{if } -3 \leq x < 0 \\ 3x + 4, & \text{if } x \geq 0 \end{cases}$ 11. $f(x) = \begin{cases} x + 3, & \text{if } x < -2 \\ x - 2, & \text{if } -2 \leq x < 2 \\ -2, & \text{if } x \geq 2 \end{cases}$

In Exercises 12–15, write the absolute value function as a piecewise function.

12. $y = |x - 3|$ 13. $y = -2|x + 4|$
 14. $y = -|x + 1| + 3$ 15. $y = 5|x - 2| + 1$