

CHAPTER 2

Polynomial Functions

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**Chapter
2****Maintaining Mathematical Proficiency****Simplify the expression.**

1. $-8x - 9x$

2. $25r - 5 + 7r - r$

3. $5 + 13t - 9 + t - 8t$

4. $4 - (a + 2)$

5. $3 + 6(3x - 5) + x$

6. $3y - (2y - 5) + 11$

7. $-3(h + 7) - 7(10 - h)$

8. $5 - 8x^2 + 5x + 8x^2$

9. $6(x^2 - 2) + x(3 - x)$

Solve the equation by factoring.

10. $x^2 + 8x + 15 = 0$

11. $x^2 + 3x - 18 = 0$

12. $x^2 - 2x - 8 = 0$

13. $x^2 + 12x = -36$

14. $2x^2 - 24 = 8x$

15. $3x^2 = 18x - 24$

16. $5x^2 + 2 = -7x$

17. $2x = 15 - 8x^2$

18. $17x - 7 = 6x^2$

2.1

Graphing Polynomial Functions

For use with Exploration 2.1

Essential Question What are some common characteristics of the graphs of cubic and quartic polynomial functions?

1 EXPLORATION: Identifying Graphs of Polynomial Functions

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Match each polynomial function with its graph. Explain your reasoning. Use a graphing calculator to verify your answers.

a. $f(x) = x^3 - x$

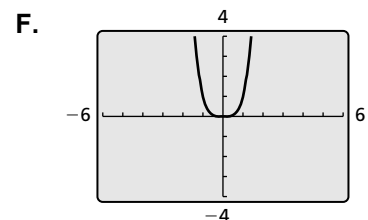
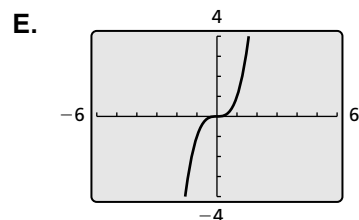
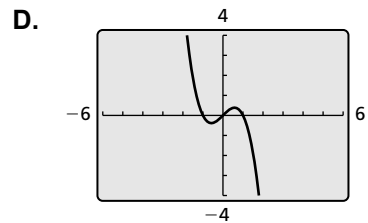
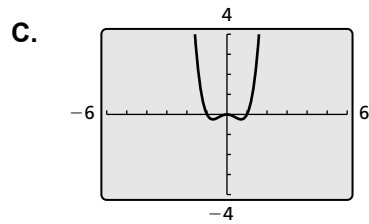
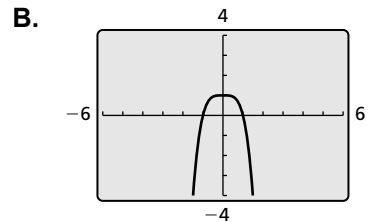
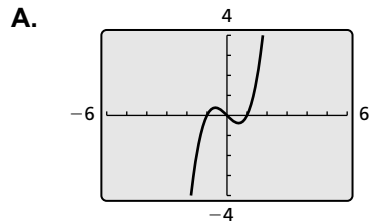
b. $f(x) = -x^3 + x$

c. $f(x) = -x^4 + 1$

d. $f(x) = x^4$

e. $f(x) = x^3$

f. $f(x) = x^4 - x^2$



2.1 Graphing Polynomial Functions (continued)**2 EXPLORATION: Identifying x -Intercepts of Polynomial Graphs**

Work with a partner. Each of the polynomial graphs in Exploration 1 has x -intercept(s) of -1 , 0 , or 1 . Identify the x -intercept(s) of each graph. Explain how you can verify your answers.

Communicate Your Answer

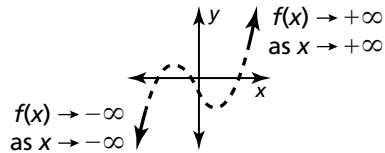
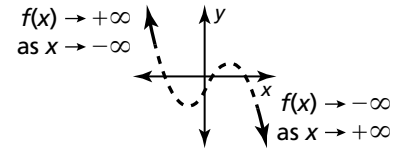
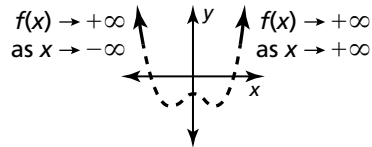
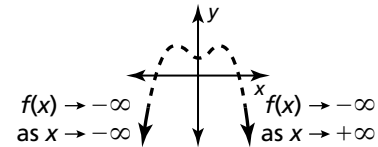
3. What are some common characteristics of the graphs of cubic and quartic polynomial functions?

4. Determine whether each statement is *true* or *false*. Justify your answer.
 - a. When the graph of a cubic polynomial function rises to the left, it falls to the right.

 - b. When the graph of a quartic polynomial function falls to the left, it rises to the right.

2.1**Practice**

For use after Lesson 2.1

Core Concepts**End Behavior of Polynomial Functions****Degree:** odd**Leading coefficient:** positive**Degree:** odd**Leading coefficient:** negative**Degree:** even**Leading coefficient:** positive**Degree:** even**Leading coefficient:** negative**Notes:****Worked-Out Examples****Example #1**Evaluate the function for the given value of x .

$$\begin{aligned} h(x) &= -3x^4 + 2x^3 - 12x - 6 \\ h(-2) &= -3(-2)^4 + 2(-2)^3 - 12(-2) - 6 \\ &= -48 - 16 + 24 - 6 \\ &= -46 \end{aligned}$$

Example #2

Describe the end behavior of the graph of the function.

$$g(x) = 7x^7 + 12x^5 - 6x^3 - 2x - 18$$

The function has degree 7 and leading coefficient 7. Because the degree is odd and the leading coefficient is positive, $g(x) \rightarrow -\infty$ as $x \rightarrow -\infty$ and $g(x) \rightarrow +\infty$ as $x \rightarrow +\infty$.

2.1 Practice (continued)**Practice A**

In Exercises 1–4, decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

1. $f(x) = 2x^2 - 3x^4 + 6x + 1$

2. $m(x) = -\frac{3}{7}x^3 + \frac{7}{x} - 3$

3. $g(x) = \sqrt{15}x + \sqrt{5}$

4. $p(x) = -2\sqrt{3} + 3x - 2x^2$

In Exercises 5 and 6, evaluate the function for the given value of x .

5. $h(x) = -x^3 - 2x^2 - 3x + 4; x = 2$

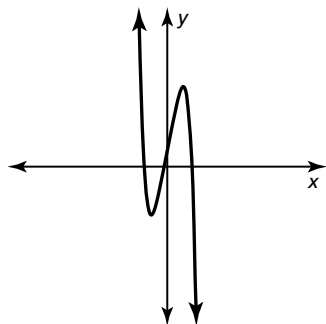
6. $g(x) = x^4 - 32x^2 + 256; x = -4$

In Exercises 7 and 8, describe the end behavior of the graph of the function.

7. $f(x) = -3x^6 + 4x^2 - 3x + 6$

8. $f(x) = \frac{4}{5}x + 6x + 3x^5 - 3x^3 - 2$

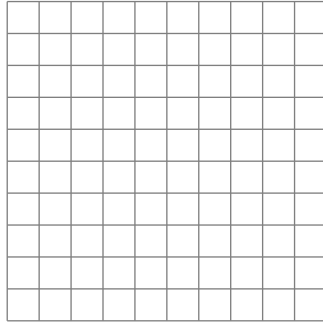
9. Describe the degree and leading coefficient of the polynomial function using the graph.



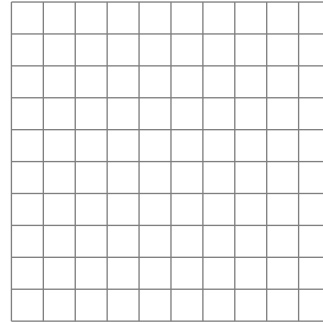
2.1 Practice (continued)

In Exercises 10 and 11, graph the polynomial function.

10. $p(x) = 16 - x^4$



11. $g(x) = x^2 + 3x^5 - x$



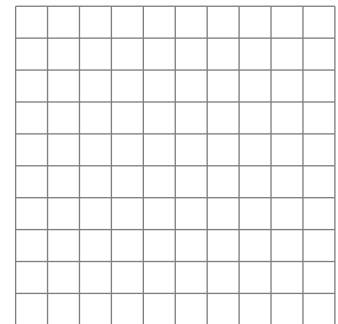
12. Sketch a graph of the polynomial function f if

f is increasing when $x < -1$ and $0 < x < 1$,

f is decreasing when $-1 < x < 0$ and $x > 1$,

and $f(x) < 0$ for all real numbers.

Describe the degree and leading coefficient of the function f .



13. The number of students S (in thousands) who graduate in four years from a university can be modeled by the function $S(t) = -\frac{1}{4}t^3 + t^2 + 23$, where t is the number of years since 2010.

a. Use a graphing calculator to graph the function for the interval $0 \leq t \leq 5$. Describe the behavior of the graph on this interval.

b. What is the average rate of change in the number of four-year graduates from 2010 to 2015?

c. Do you think this model can be used for years before 2010 or after 2015? Explain your reasoning.

Practice B

In Exercises 1–4, decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

1. $h(x) = 6x^3 - 9x^{-3} + x^2 - 5x - 1$

2. $f(x) = 11x^2 - \sqrt{7} + 12x$

3. $g(x) = 2x^4 - \frac{1}{3}x^2 - \sqrt{14}x^3 + 2x - \frac{5}{3}$

4. $f(x) = 2x^3 + 9x^2 - 5x + \frac{4}{x} - 1$

In Exercises 5–7, evaluate the function for the given value of x .

5. $f(x) = -x^3 + 5x^2 + 9x + 4$; $x = -11$

6. $g(x) = 3x^3 + 6x^2 + 12x - 10$; $x = \frac{1}{3}$

7. $h(x) = 9x^3 - 8x^2 + 11x + 8$; $x = -\frac{1}{2}$

In Exercises 8 and 9, describe the end behavior of the graph of the function.

8. $g(x) = -5x^4 + 7x^3 - 7x^6 + x^2 - 9x + 2$

9. $h(x) = -2x^3 + 5x^2 + 4x^5 - 3x^4 + 12x^2 - 4$

In Exercises 10–13, graph the polynomial function.

10. $q(x) = x^4 - x^3 - 5x^2$

11. $h(x) = 4 - 2x^2 - x^4$

12. $k(x) = x^5 - 2x^4 + x - 2$

13. $f(x) = x^6 - 3x^5 + 2x^3 + x + 1$

In Exercises 14 and 15, sketch a graph of the polynomial function f having the given characteristics. Use the graph to describe the degree and leading coefficient of the function f .

14. f is increasing when $x < 1$; f is decreasing when $x > 1$.

$$f(x) > 0 \text{ when } -1 < x < 3; f(x) < 0 \text{ when } x < -1 \text{ and } x > 3.$$

15. f is increasing when $x < -1.1$ and $x > 2.4$; f is decreasing when $-1.1 < x < 2.4$.

$$f(x) > 0 \text{ when } -2 < x < 0 \text{ and } x > 4; f(x) < 0 \text{ when } x < -2 \text{ and } 0 < x < 4.$$

16. The function $h(t) = -4.9t^2 + 28.62t + 2.4$ models the height h of a high pop-up hit by a baseball player after t seconds. Use a graphing calculator to graph the function. State an appropriate window to view the maximum height of the ball and when the ball hits the ground.