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2.2 Dividing Polynomials
For use with Exploration 2.2

Essential Question How can you use the factors of a cubic polynomial to solve a division problem involving the polynomial?

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1 EXPLORATION: Dividing Polynomials

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Match each division statement with the graph of the related cubic polynomial $f(x)$. Explain your reasoning. Use a graphing calculator to verify your answers.
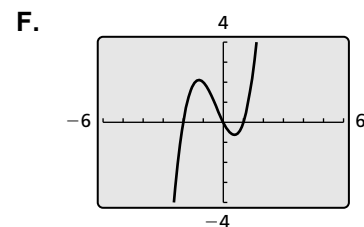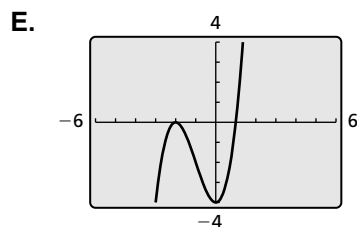
a. $\dfrac{f(x)}{x} = (x-1)(x+2)$

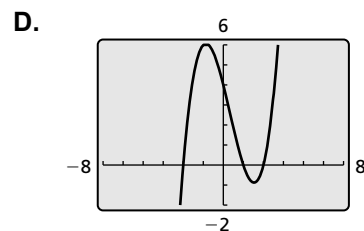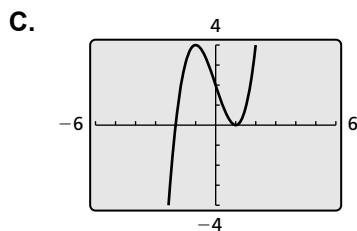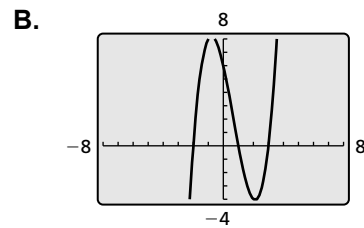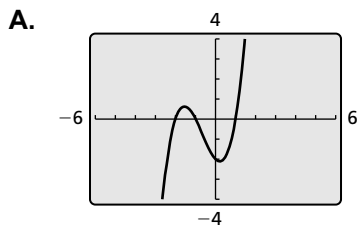**b.** $\dfrac{f(x)}{x-1} = (x-1)(x+2)$

c. $\dfrac{f(x)}{x+1} = (x-1)(x+2)$

d. $\dfrac{f(x)}{x-2} = (x-1)(x+2)$

e. $\dfrac{f(x)}{x+2} = (x-1)(x+2)$

f. $\dfrac{f(x)}{x-3} = (x-1)(x+2)$

A.

B.

C.

D.

E.

F.

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2 **EXPLORATION:** Dividing Polynomials

Work with a partner. Use the results of Exploration 1 to find each quotient. Write your answers in standard form. Check your answers by multiplying.

a. $\left(x^3 + x^2 - 2x\right) \div x$

b. $\left(x^3 - 3x + 2\right) \div (x - 1)$

c. $\left(x^3 + 2x^2 - x - 2\right) \div (x + 1)$

d. $\left(x^3 - x^2 - 4x + 4\right) \div (x - 2)$

e. $\left(x^3 + 3x^2 - 4\right) \div (x + 2)$

f. $\left(x^3 - 2x^2 - 5x + 6\right) \div (x - 3)$

Communicate Your Answer

3. How can you use the factors of a cubic polynomial to solve a division problem involving the polynomial?

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Core Concepts

The Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

Notes:

Worked-Out Examples

Example #1

Divide using polynomial long division.

$(3x^2 - 14x - 5) \div (x - 5)$

$$
\begin{array}{r}
3x + 1 \\
x - 5 \overline{)\,3x^2 - 14x - 5} \\
\underline{3x^2 - 15x} \\
x - 5 \\
\underline{x - 5} \\
0
\end{array}
$$

$(3x^2 - 14x - 5) \div (x - 5) = 3x + 1$

Example #2

Divide using synthetic division.

$(x^3 - 4x + 6) \div (x + 3)$

Use synthetic division. Because the divisor is
$x + 3 = x - (-3), k = -3.$

$$
\begin{array}{r|rrrr}
-3 & 1 & 0 & -4 & 6 \\
   &   & -3 & 9 & -15 \\
\hline
   & 1 & -3 & 5 & -9
\end{array}
$$

$$\frac{x^3 - 4x + 6}{x + 3} = x^2 - 3x + 5 - \frac{9}{x + 3}$$

2.2 **Practice** (continued)

Practice A

In Exercises 1–4, divide using polynomial long division.

1. $\left(x^2 + 6x + 12\right) \div (x - 3)$

2. $\left(x^3 - 4x^2\right) \div \left(x^2 - 16\right)$

3. $\left(4x^3 + 13x^2 + 27x + 6\right) \div \left(4x + 1\right)$

4. $\left(x^4 + 2x^3 + 5x^2 + 3x\right) \div \left(x^2 - x\right)$

In Exercises 5–8, divide using synthetic division.

5. $\left(x^2 - 10x + 2\right) \div (x - 2)$

6. $\left(x^3 + 4x^2 + 6x + 4\right) \div (x + 2)$

7. $\left(2x^3 - 54\right) \div (x + 3)$

8. $\left(2x^4 - 11x^3 + 11x^2 + 4x + 4\right) \div (x - 4)$

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In Exercises 9–12, match the equivalent expressions. Justify your answers.

9. $\left(x^2 - x - 8\right) \div \left(x - 4\right)$ **A.** $x + 3 + \dfrac{4}{x - 4}$

10. $\left(x^2 - x + 8\right) \div \left(x - 4\right)$ **B.** $x + 5 + \dfrac{12}{x - 4}$

11. $\left(x^2 + x - 8\right) \div \left(x - 4\right)$ **C.** $x + 5 + \dfrac{28}{x - 4}$

12. $\left(x^2 + x + 8\right) \div \left(x - 4\right)$ **D.** $x + 3 + \dfrac{20}{x - 4}$

In Exercises 13–16, use synthetic division to evaluate the function for the indicated value of *x*.

13. $f(x) = -3x^3 + 4x^2 - 17x - 6;\ x = 2$ **14.** $f(x) = -x^4 + x^2 + 4;\ x = -1$

15. $f(x) = x^3 - 10x^2 + 31x - 30;\ x = -5$ **16.** $f(x) = x^3 + 8x + 27;\ x = 3$

17. What is the value of k such that $\left(-x^4 + 5x^2 + kx - 8\right) \div \left(x - 4\right)$ has a remainder of 0?

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Practice B

In Exercises 1–3, divide using polynomial long division.

1. $\left(x^3 + 3x^2 - 4x - 6\right) \div \left(x^2 - 4\right)$

2. $\left(4x^4 + 2x^3 - 9x^2 - 36\right) \div \left(x^2 + x - 4\right)$

3. $\left(2x^4 - 40x^2 - 28\right) \div \left(x^2 - 5x - 2\right)$

In Exercises 4–9, divide using synthetic division.

4. $\left(4x^2 - 15x + 7\right) \div (x - 2)$

5. $\left(x^3 - 9x + 12\right) \div (x + 3)$

6. $\left(x^2 + 16\right) \div (x - 4)$

7. $\left(2x^3 - 5x^2 + 3\right) \div (x + 1)$

8. $\left(x^4 + 5x^3 - 6x^2 - 11x + 14\right) \div (x + 4)$

9. $\left(x^4 + 2x^3 + 4x - 20\right) \div (x + 6)$

10. Describe and correct the error in using synthetic division to divide
$x^3 + 2x^2 + 7$ by $x + 3$.

$$
\begin{array}{r|rrrr}
\diagdown \quad -3 & 1 & 2 & 0 & 7 \\
 & & -3 & 3 & -9 \\
\hline
 & 1 & -1 & 3 & -2
\end{array}
$$

$$\frac{x^3 + 2x^2 + 7}{x + 3} = x^3 - x^2 + 3x - 2$$

In Exercises 11–14, use synthetic division to evaluate the function for the indicated value of *x*.

11. $f(x) = x^3 + x^2 - 4x + 3;\ x = -1$

12. $f(x) = -x^3 - 6x^2 + 6;\ x = -2$

13. $f(x) = x^4 + 5x^2 - 8x + 1;\ x = 4$

14. $f(x) = -x^4 - x^2 - 5;\ x = 3$

15. What is the value of k such that $\left(x^3 + kx^2 - 9x - 36\right) \div (x + 4)$ has a remainder of zero?