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2.3 Factoring Polynomials

For use with Exploration 2.3

Essential Question How can you factor a polynomial?

1 EXPLORATION: Factoring Polynomials

Work with a partner. Match each polynomial equation with the graph of its related polynomial function. Use the x-intercepts of the graph to write each polynomial in factored form. Explain your reasoning.

a. $x^2 + 5x + 4 = 0$

b. $x^3 - 2x^2 - x + 2 = 0$

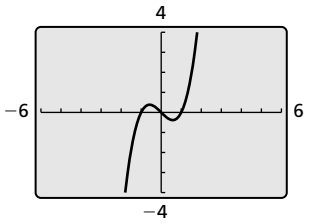
c. $x^3 + x^2 - 2x = 0$

d. $x^3 - x = 0$

e. $x^4 - 5x^2 + 4 = 0$

f. $x^4 - 2x^3 - x^2 + 2x = 0$

A.



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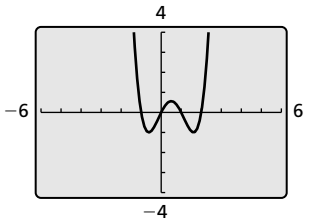

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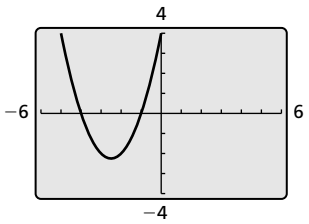

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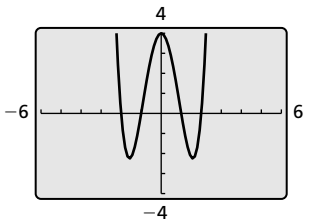
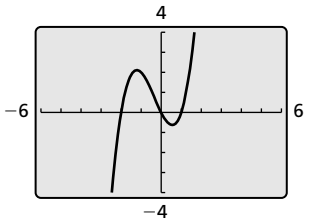
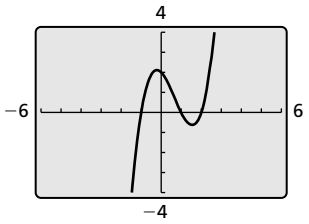


E.



F.



2 EXPLORATION: Factoring Polynomials

Work with a partner. Use the x-intercepts of the graph of the polynomial function to write each polynomial in factored form. Explain your reasoning. Check your answers by multiplying.

a. $f(x) = x^2 - x - 2$

b. $f(x) = x^3 - x^2 - 2x$

c. $f(x) = x^3 - 2x^2 - 3x$

d. $f(x) = x^3 - 3x^2 - x + 3$

e. $f(x) = x^4 + 2x^3 - x^2 - 2x$

f. $f(x) = x^4 - 10x^2 + 9$

Communicate Your Answer

3. How can you factor a polynomial?

4. What information can you obtain about the graph of a polynomial function written in factored form?

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2.3 Practice

For use after Lesson 2.3

Core Concepts

Special Factoring Patterns

Sum of Two Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Example

$$64x^3 + 1 = (4x)^3 + 1^3$$
$$= (4x + 1)(16x^2 - 4x + 1)$$

Difference of Two Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example

$$27x^3 - 8 = (3x)^3 - 2^3$$
$$= (3x - 2)(9x^2 + 6x + 4)$$

Notes:

The Factor Theorem

A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

Notes:

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2.3 Practice (continued)

Worked-Out Examples

Example #1

Factor the polynomial completely.

$$x^3 - 2x^2 - 24x = x(x^2 - 2x - 24)$$
$$= x(x - 6)(x + 4)$$

Example #2

Factor the polynomial completely.

$$3r^8 + 3r^5 - 60r^2 = 3r^2(r^6 + r^3 - 20)$$
$$= 3r^2(r^3 + 5)(r^3 - 4)$$

Practice A

In Exercises 1–14, factor the polynomial completely.

1. $20x^3 - 220x^2 + 600x$

2. $m^5 - 81m$

3. $27a^3 + 8b^3$

4. $5t^6 + 2t^5 - 5t^4 - 2t^3$

5. $y^4 - 13y^2 - 48$

6. $5p^3 + 5p - 5p^2 - 5$

7. $810k^4 - 160$

8. $a^5 + a^3 - a^2 - 1$

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2.3 Practice (continued)

9. $2x^6 - 8x^5 - 42x^4$

10. $5z^3 + 5z^2 - 6z - 6$

11. $12x^2 - 22x - 20$

12. $3m^2 - 48m^6$

13. $4x^3 - 4x^2 + x$

14. $5m^4 - 70m^3 + 245m^2$

In Exercises 15–17, show that the binomial is a factor of $f(x)$. Then factor $f(x)$ completely.

15. $f(x) = x^3 - 13x - 12;\ x + 1$

16. $f(x) = 6x^3 + 8x^2 - 34x - 12;\ x - 2$

17. $f(x) = 2x^4 - 12x^3 + 6x^2 + 20x;\ x - 5$

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Practice B

In Exercises 1–6, factor the polynomial completely.

1. $5t^5 - 320t^3$

2. $2p^6 - 26p^5 + 84p^4$

3. $3x^4 - 432x^2$

4. $5a^6 - 16a^5 - 45a^4$

5. $12j^9 - 28j^8 + 15j^7$

6. $15q^{10} + 38q^9 + 24q^8$

In Exercises 7–9, factor the polynomial completely.

7. $2p^9 - 16p^6$

8. $25k^8 + 1600k^5$

9. $54w^7 - 16w^4$

In Exercises 10–13, factor the polynomial completely.

10. $x^3 - 7x^2 + 5x - 35$

11. $m^3 - 2m^2 - 16m + 32$

12. $9w^3 - 27w^2 - 4w + 12$

13. $25s^3 + 100s^2 - s - 4$

In Exercises 14–16, factor the polynomial completely.

14. $81g^4 - 625$

15. $2t^8 + 6t^5 - 20t^2$

16. $5v^{10} - 25v^6 + 30v^2$

In Exercises 17–20, determine whether the binomial is a factor of $f(x)$.

17. $f(x) = 4x^3 - 15x^2 - 30x + 25;\ x - 5$

18. $f(x) = 2x^3 + 16x^2 - 4x - 50;\ x + 7$

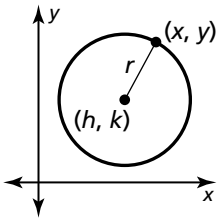
19. $f(x) = 8x^5 + 43x^4 - 58x^3 + 60x^2 - 70;\ x - 4$

20. $f(x) = 42x^4 + 143x^3 + 37x^2 - 27x + 45;\ x - 2$

21. Fill in the blank of the divisor so that the remainder is 0. Justify your answer.

$f(x) = 2x^3 + 7x^2 - 4x;\ (x + \_\_\_)$

22. The standard equation of a circle with radius r and center (h, k) is $(x - h)^2 + (y - k)^2 = r^2$. Rewrite the equation of each circle in standard form. Identify the center and radius of the circle. Then graph the circle.



a. $x^2 + 8x + 16 + y^2 = 9$

b. $x^2 - 10x + 25 + y^2 = 4$

c. $x^2 - 4x + 4 + y^2 + 6y + 9 = 16$