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2.4 Solving Polynomial Equations

For use with Exploration 2.4

Essential Question

How can you determine whether a polynomial equation has a repeated solution?

1 EXPLORATION: Cubic Equations and Repeated Solutions

Work with a partner. Some cubic equations have three distinct solutions. Others have repeated solutions. Match each cubic polynomial equation with the graph of its related polynomial function. Then solve each equation. For those equations that have repeated solutions, describe the behavior of the related function near the repeated zero using the graph or a table of values.

a. $x^3 - 6x^2 + 12x - 8 = 0$

b. $x^3 + 3x^2 + 3x + 1 = 0$

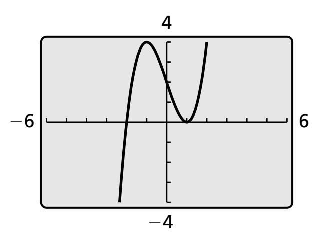
c. $x^3 - 3x + 2 = 0$

d. $x^3 + x^2 - 2x = 0$

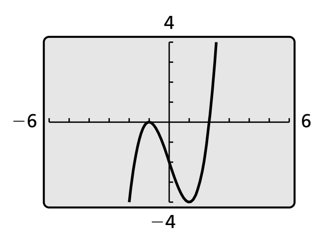
e. $x^3 - 3x - 2 = 0$

f. $x^3 - 3x^2 + 2x = 0$

A.



B.



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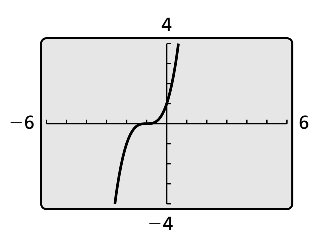

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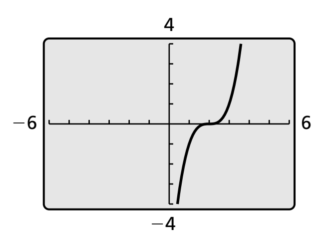
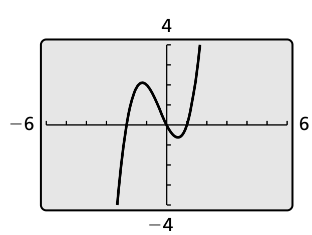
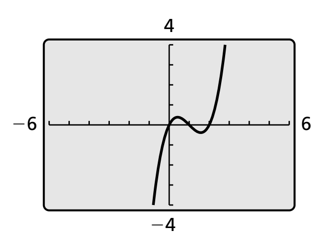


E.



F.



2 EXPLORATION: Quartic Equations and Repeated Solutions

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Determine whether each quartic equation has repeated solutions using the graph of the related quartic function or a table of values. Explain your reasoning. Then solve each equation.

a. $x^4 - 4x^3 + 5x^2 - 2x = 0$

b. $x^4 - 2x^3 - x^2 + 2x = 0$

c. $x^4 - 4x^3 + 4x^2 = 0$

d. $x^4 + 3x^3 = 0$

Communicate Your Answer

3. How can you determine whether a polynomial equation has a repeated solution?

4. Write a cubic or a quartic polynomial equation that is different from the equations in Explorations 1 and 2 and has a repeated solution.

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2.4 Practice

For use after Lesson 2.4

Core Concepts

The Rational Root Theorem

If $f(x) = a_n x^n + \cdots + a_1 x + a_0$ has *integer* coefficients, then every rational solution of $f(x) = 0$ has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Notes:

The Irrational Conjugates Theorem

Let f be a polynomial function with rational coefficients, and let a and b be rational numbers such that $\sqrt{b}$ is irrational. If $a + \sqrt{b}$ is a zero of f, then $a - \sqrt{b}$ is also a zero of f.

Notes:

Worked-Out Examples

Example #1

Solve the equation.

$$2c^4 - 6c^3 = 12c^2 - 36c$$
$$2c^4 - 6c^3 - 12c^2 + 36c = 0$$
$$2c(c^3 - 3c^2 - 6c + 18) = 0$$
$$2c[c^2(c - 3) - 6(c - 3)] = 0$$
$$2c[(c^2 - 6)(c - 3)] = 0$$
$$2c\left(c - \sqrt{6}\right)\left(c + \sqrt{6}\right)(c - 3) = 0$$

The solutions are $c = -\sqrt{6}$, $c = 0$, $c = \sqrt{6}$, and $c = 3$.

Example #2

Find the zeros of the function. Then sketch a graph of the function.

$x^3 - 16x^2 + 55x + 72 = 0$

Step 1 The possible rational solutions are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 9, \pm 12, \pm 18, \pm 24, \pm 36, \pm 72$.

Step 2

−1	1	−16	55	72
		−1	17	−72
	1	−17	72	0

So, $x + 1$ is a factor.

Step 3

$$x^3 - 16x^2 + 55x + 72 = 0$$
$$(x + 1)(x^2 - 17x + 72) = 0$$
$$(x + 1)(x - 9)(x - 8) = 0$$

So, the real solutions are $x = -1$, $x = 8$, and $x = 9$.

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2.4 **Practice (continued)**

Practice A

In Exercises 1–6, solve the equation.

1. $36r^3 - r = 0$

2. $20x^3 + 80x^2 = -60x$

3. $3m^2 = 75m^4$

4. $-13y^2 + 36 = -y^4$

5. $2x^3 - x^2 - 2x = -1$

6. $-20c^2 + 50c = 8c^3 - 125$

In Exercises 7–10, find the zeros of the function. Then sketch a graph of the function.

7. $f(x) = x^4 - x^3 - 12x^2$

8. $f(x) = -4x^3 + 12x^2 - 9x$

9. $f(x) = x^3 + 4x^2 - 6x - 24$

10. $f(x) = x^4 - 18x^2 + 81$

11. According to the Rational Root Theorem, which is *not* a possible solution of the equation $2x^4 + 3x^3 - 6x + 7 = 0$?

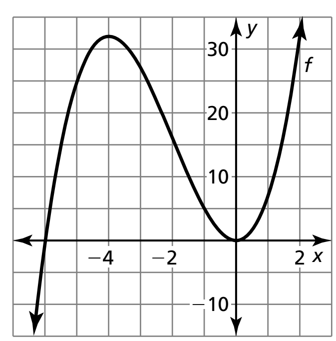
A. 3.5 **B.** 0.5 **C.** 7 **D.** 2

12. Find all the real zeros of the function $f(x) = 3x^4 + 11x^3 - 40x^2 - 132x + 48$.

13. Write a polynomial function g of least degree that has rational coefficients, a leading coefficient of 1, and the zeros -5 and $4 + \sqrt{2}$.

14. Use the information in the graph to answer the questions.

a. What are the real zeros of the function f?



b. Write an equation of the cubic function in factored form.

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Practice B

In Exercises 1–6, solve the equation.

1. $4x^4 + 12x^3 + 9x^2 = 0$

2. $6h^5 = 12h^3$

3. $16q^4 - 8q^2 + 1 = 0$

4. $w^4 + 81 = 18w^2$

5. $p^3 - 25p = 50 - 2p^2$

6. $y^3 - 8y^2 = 9y - 72$

In Exercises 7–10, find the zeros of the function. Then sketch a graph of the function.

7. $f(x) = -5x^4 + 20x^3 + 60x^2$

8. $g(x) = -x^3 - x^2 + 30x$

9. $h(x) = x^3 + x^2 - 4x - 4$

10. $f(x) = x^3 - 4x^2 - 9x + 36$

11. According to the Rational Root Theorem, which is *not* a possible zero of the function $f(x) = 24x^4 - 16x^3 + 21x - 27$?

A. $-\frac{3}{8}$ **B.** -2 **C.** $-\frac{1}{3}$ **D.** $-\frac{9}{4}$

12. Describe and correct the error in listing the possible rational zeros of the function.

> ✗ $f(x) = 2x^3 + 5x^2 - 2x - 6$
>
> Possible zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

In Exercises 13 and 14, find all the real solutions of the equation.

13. $2x^3 - 3x^2 + 18x - 27 = 0$

14. $x^3 - 5x^2 - 2x + 24 = 0$

15. Write a third or fourth degree polynomial function that has zeros at $\pm\frac{7}{5}$. Justify your answer.

16. The sidewalk hazard marker is shaped like a pyramid, with a height 2 centimeters greater than the length of each side of its square base. The volume of the marker is 297 cubic centimeters. What are the dimensions of the sidewalk hazard marker?