Name____ Date__________

2.5 The Fundamental Theorem of Algebra

For use with Exploration 2.5

Essential Question

How can you determine whether a polynomial equation has imaginary solutions?

1 EXPLORATION: Cubic Equations and Imaginary Solutions

Work with a partner. Match each cubic polynomial equation with the graph of its related polynomial function. Then find *all* solutions. Make a conjecture about how you can use a graph or table of values to determine the number and types of solutions of a cubic polynomial equation.

a. $x^3 - 3x^2 + x + 5 = 0$

b. $x^3 - 2x^2 - x + 2 = 0$

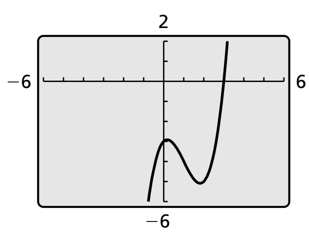
c. $x^3 - x^2 - 4x + 4 = 0$

d. $x^3 + 5x^2 + 8x + 6 = 0$

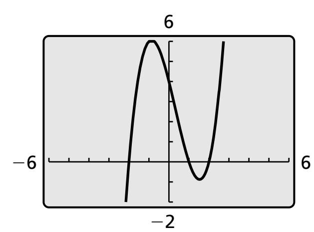
e. $x^3 - 3x^2 + x - 3 = 0$

f. $x^3 - 3x^2 + 2x = 0$

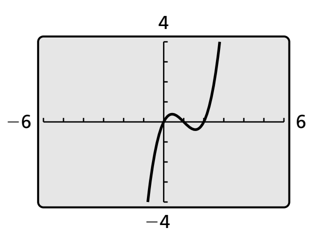
A.



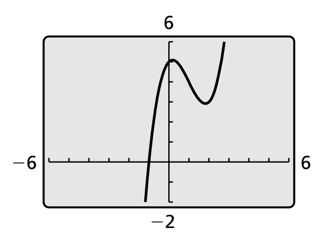
B.



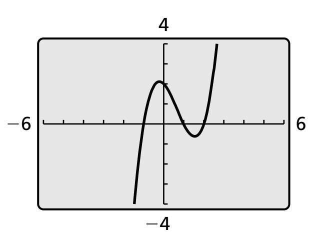
C.



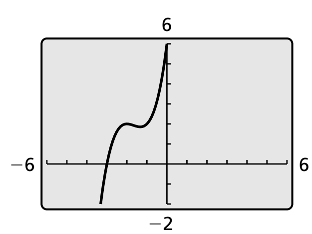
D.



E.



F.



2 EXPLORATION: Quartic Equations and Imaginary Solutions

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use the graph of the related quartic function, or a table of values, to determine whether each quartic equation has imaginary solutions. Explain your reasoning. Then find *all* solutions.

a. $x^4 - 2x^3 - x^2 + 2x = 0$

b. $x^4 - 1 = 0$

c. $x^4 + x^3 - x - 1 = 0$

d. $x^4 - 3x^3 + x^2 + 3x - 2 = 0$

Communicate Your Answer

3. How can you determine whether a polynomial equation has imaginary solutions?

4. Is it possible for a cubic equation to have three imaginary solutions? Explain your reasoning.

Name____ Date __________

2.5 Practice
For use after Lesson 2.5

Core Concepts

The Fundamental Theorem of Algebra

Theorem If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has at least one solution in the set of complex numbers.

Corollary If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has exactly n solutions provided each solution repeated twice is counted as two solutions, each solution repeated three times is counted as three solutions, and so on.

Notes:

The Complex Conjugates Theorem

If f is a polynomial function with real coefficients, and $a + bi$ is an imaginary zero of f, then $a - bi$ is also a zero of f.

Notes:

Name ______________________________ Date _________

2.5 **Practice (continued)**

Descartes's Rule of Signs

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ be a polynomial function with real coefficients.

- The number of *positive real zeros* of *f* is equal to the number of changes in sign of the coefficients of $f(x)$ or is less than this by an even number.
- The number of *negative real zeros* of *f* is equal to the number of changes in sign of the coefficients of $f(-x)$ or is less than this by an even number.

Notes:

Worked-Out Examples

Example #1

Identify the number of solutions of the polynomial equation. Then find all solutions of the equation.

$z^4 + 5z^2 - 14 = 0$

Because $z^4 + 5z^2 - 14 = 0$ is a polynomial equation of degree 4, it has four solutions. Solve by factoring.

$$z^4 + 5z^2 - 14 = 0$$
$$(z^2 - 2)(z^2 + 7) = 0$$
$$z^2 - 2 = 0 \quad or \quad z^2 + 7 = 0$$
$$z^2 = 2 \quad or \quad z^2 = -7$$
$$z = \pm\sqrt{2} \quad or \quad z = \pm\sqrt{-7}$$
$$z = \pm\, i\sqrt{7}$$

The solutions are $-\sqrt{2}$, $\sqrt{2}$, $-i\sqrt{7}$, and $i\sqrt{7}$.

Example #2

Find all zeros of the polynomial function.

$f(x) = x^4 + 5x^3 - 7x^2 - 29x + 30$

Find the rational zeros of *f*. Because *f* is a polynomial function of degree 4, it has four zeros. The possible rational zeros are ±1, ±2, ±3, ±5, ±6, ±10, ±15, and ±30. Using synthetic division, you can determine that -5, -3, 1, and 2 are zeros.

Name______________________________ Date__________

2.5 **Practice (continued)**

Practice A

In Exercises 1–4, find all zeros of the polynomial function.

1. $h(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$

2. $f(x) = x^3 - 3x^2 - 15x + 125$

3. $g(x) = x^4 - 48x^2 - 49$

4. $h(x) = -5x^3 + 9x^2 - 18x - 4$

In Exercises 5–8, write a polynomial function *f* of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.

5. $-4, 1, 7$

6. $10, -\sqrt{5}$

7. $8, 3 - i$

8. $0, 2 - \sqrt{2}, 2 + 3i$

Name ______________________________ Date __________

Practice B

In Exercises 1–4, identify the number of solutions of the polynomial equation. Then find all solutions of the equation.

1. $8x^3 + 27 = 0$

2. $4p^5 - 32p^2 = 0$

3. $t^8 - t^4 - t^2 + 1 = 0$

4. $x^5 - 9x^3 + 8x^2 - 72 = 0$

In Exercises 5–8, find all zeros of the polynomial function.

5. $h(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$

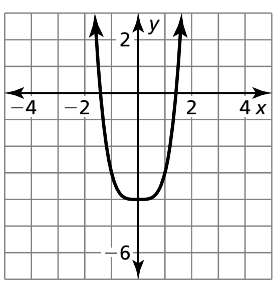
6. $f(x) = x^4 - 12x^3 + 54x^2 - 108x + 81$

7. $g(x) = x^5 + 4x^4 + x^3 - 14x^2 - 20x - 8$

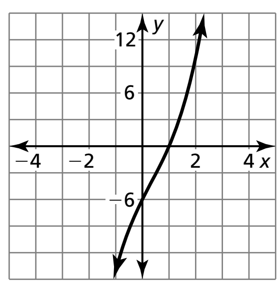
8. $f(x) = x^5 + 2x^4 - 13x^3 - 26x^2 + 36x + 72$

In Exercises 9 and 10, determine the number of imaginary zeros for the function with the given degree and graph. Explain your reasoning.

9. Degree: 4



10. Degree: 3



In Exercises 11–13, write a polynomial function *f* of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.

11. $2,\ 3 + i$

12. $2i,\ 1 - i$

13. $3,\ -\sqrt{7}$

14. Two zeros of $f(x) = x^3 - 2x^2 + 9x - 18$ are $3i$ and $-3i$. Explain why the third zero must be a real number.

15. Use Descartes' Rule of Signs to determine which function has no positive real zeros.

A. $f(x) = x^4 - 3x^2 + 6x - 7$

B. $f(x) = x^4 + 2x^2 + 4x - 3$

C. $f(x) = x^4 + x^2 + 10$

D. $f(x) = x^4 + 5x^3 - 9x - 7$