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# 2.6 Transformations of Polynomial Functions
**For use with Exploration 2.6**

**Essential Question** How can you transform the graph of a polynomial function?

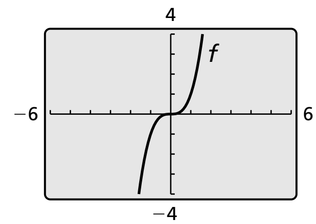
## 1 EXPLORATION: Transforming the Graph of a Cubic Function

**Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.**

**Work with a partner.** The graph of the cubic function

$$f(x) = x^3$$

is shown. The graph of each cubic function $g$ represents a transformation of the graph of $f$. Write a rule for $g$. Use a graphing calculator to verify your answers.

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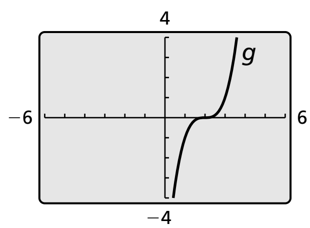


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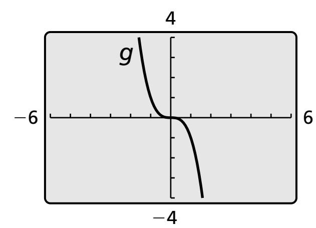


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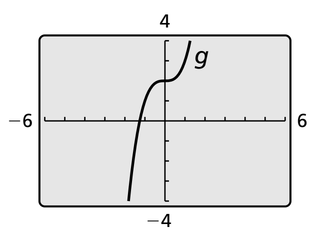
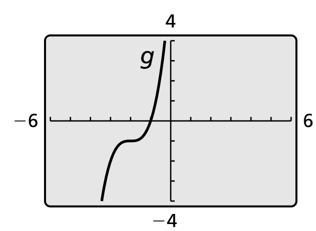



**d.**



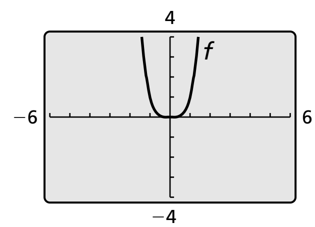
## 2 EXPLORATION: Transforming the Graph of a Quartic Function

**Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.**

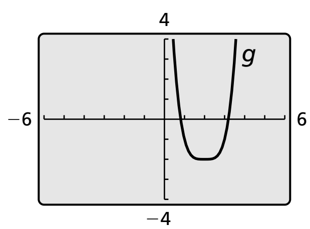
**Work with a partner.** The graph of the quartic function

$f(x) = x^4$

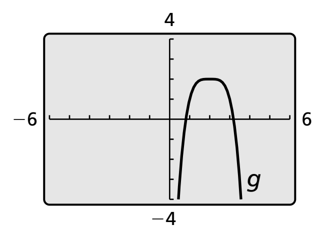
is shown. The graph of each quartic function $g$ represents a transformation of the graph of $f$. Write a rule for $g$. Use a graphing calculator to verify your answers.




**a.**



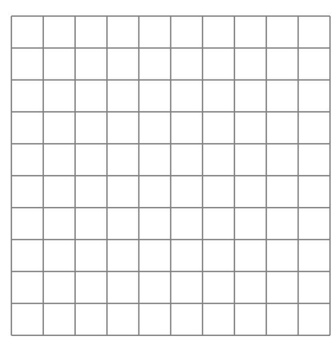

**b.**




## Communicate Your Answer

**3.** How can you transform the graph of a polynomial function?

**4.** Describe the transformation of $f(x) = x^4$ represented by $g(x) = (x + 1)^4 + 3$. Then graph $g$.



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# 2.6 Practice
**For use after Lesson 2.6**

## Core Concepts

| Transformation | $f(x)$ Notation | Examples | |
|---|---|---|---|
| **Horizontal Translation**<br>Graph shifts left or right. | $f(x-h)$ | $g(x) = (x-5)^4$<br>$g(x) = (x+2)^4$ | 5 units right<br>2 units left |
| **Vertical Translation**<br>Graph shifts up or down. | $f(x)+k$ | $g(x) = x^4 + 1$<br>$g(x) = x^4 - 4$ | 1 unit up<br>4 units down |
| **Reflection**<br>Graph flips over $x$- or $y$-axis. | $f(-x)$<br>$-f(x)$ | $g(x) = (-x)^4 = x^4$<br>$g(x) = -x^4$ | over $y$-axis<br>over $x$-axis |
| **Horizontal Stretch or Shrink**<br>Graph stretches away from or shrinks toward $y$-axis | $f(ax)$ | $g(x) = (2x)^4$<br>$g(x) = \left(\frac{1}{2}x\right)^4$ | shrink by a factor of $\frac{1}{2}$<br>stretch by a factor of 2 |
| **Vertical Stretch or Shrink**<br>Graph stretches away from or shrinks toward $x$-axis. | $a \bullet f(x)$ | $g(x) = 8x^4$<br>$g(x) = \frac{1}{4}x^4$ | stretch by a factor of 8<br>shrink by a factor of $\frac{1}{4}$ |

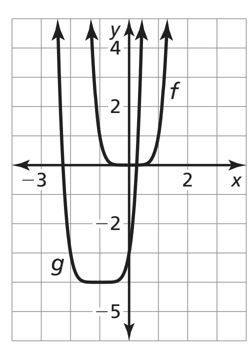
**Notes:**

## Worked-Out Examples

### Example #1

**Describe the transformation of f represented by g. Then graph each function.**

Notice that the function is of the form $g(x) = (x-h)^6 + k$. Rewrite the function to identify $h$ and $k$, $g(x) = (x-(-1))^6 + (-4)$. Because $h = -1$ and $k = -4$, the graph of $g$ is a translation 1 unit left and 4 units down of the graph of $f$.



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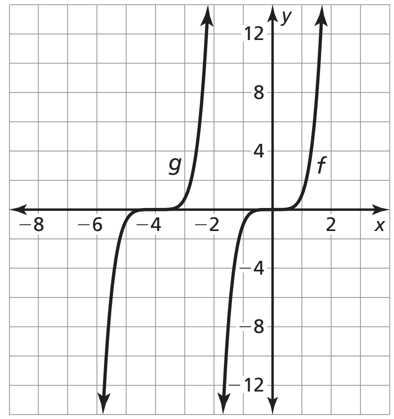
### Example #2

**Describe the transformation of f represented by g. Then graph each function**

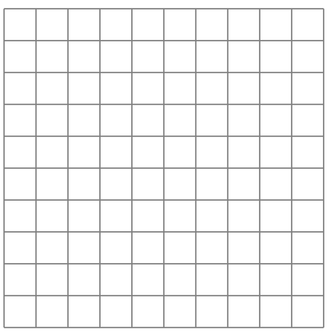
Notice that the function is of the form $g(x) = a(x - h)^5$, where $a = \frac{3}{4}$ and $h = -4$. So, the graph of $g$ is a vertical shrink by factor of $\frac{3}{4}$ followed by a translation 4 units left of the graph of $f$.



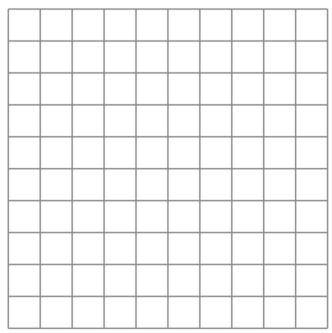

## Practice A

**In Exercises 1–6, describe the transformation of *f* represented by *g*. Then graph each function.**

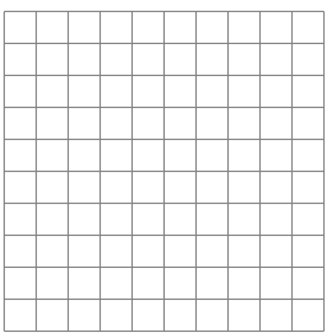
**1.** $f(x) = x^4;\ g(x) = x^4 - 9$



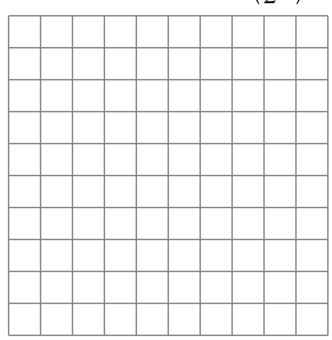
**2.** $f(x) = x^5;\ g(x) = (x + 1)^5 + 2$



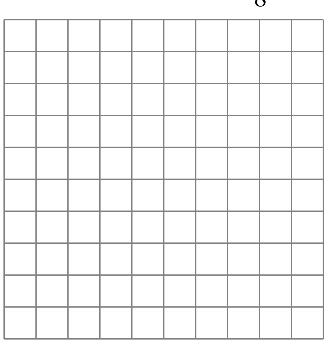
**3.** $f(x) = x^6;\ g(x) = -5(x - 2)^6$



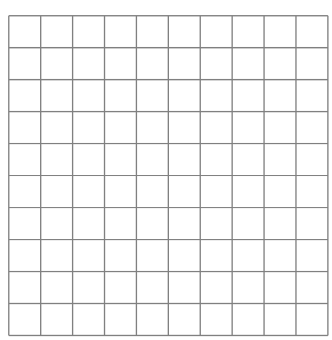
**4.** $f(x) = x^3;\ g(x) = \left(\frac{1}{2}x\right)^3 - 4$



**5.** $f(x) = x^4;\ g(x) = \frac{1}{8}(-x)^4$



**6.** $f(x) = x^5;\ g(x) = (x - 10)^5 + 1$



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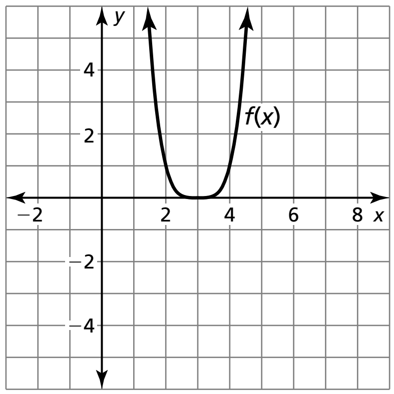
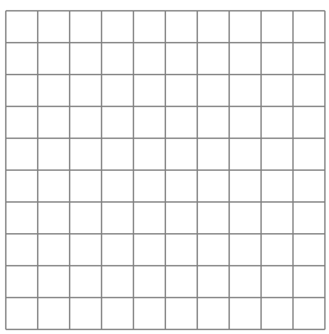
## 2.6 Practice (continued)

**7.** Graph the function $g(x) = -f(x - 3)$ on the same coordinate plane as $f(x)$.

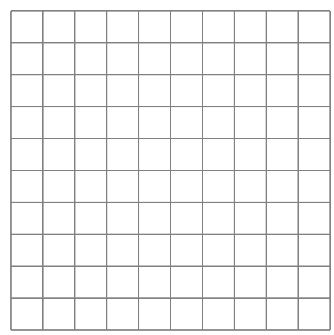



**In Exercises 8 and 9, write a rule for *g* and then graph each function. Describe the graph of *g* as a transformation of the graph of *f*.**

**8.** $f(x) = x^3 + 8; g(x) = f(-x) - 9$



**9.** $f(x) = 2x^5 - x^3 + 1; g(x) = 5f(x)$



**In Exercises 10 and 11, write a rule for *g* that represents the indicated transformations of the graph of *f*.**

**10.** $f(x) = x^3 - 6x^2 + 5$; translation 1 unit left, followed by a reflection in the $x$-axis and a vertical stretch by a factor of 2

**11.** $f(x) = 3x^4 + x^3 + 3x^2 + 12$; horizontal shrink by a factor of $\frac{1}{3}$ and a translation 8 units down, followed by a reflection in the $y$-axis

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# Practice B

**In Exercises 1 and 2, describe the transformation of *f* represented by *g*. Then graph each function.**

**1.** $f(x) = x^4,\ g(x) = (x - 3)^4 - 2$

**2.** $f(x) = x^5,\ g(x) = (x - 1)^5 + 4$

**In Exercises 3–6, describe the transformation of *f* represented by *g*. Then graph each function.**

**3.** $f(x) = x^5,\ g(x) = -3x^5$

**4.** $f(x) = x^4,\ g(x) = 3x^4 + 2$

**5.** $f(x) = x^4,\ g(x) = \frac{1}{3}x^4 - 3$

**6.** $f(x) = x^4,\ g(x) = \frac{2}{3}(x + 3)^4$

**In Exercises 7 and 8, write a rule for *g* and then graph each function. Describe the graph of *g* as a transformation of the graph of *f*.**

**7.** $f(x) = x^3 - 4x^2 + 2,\ g(x) = -\frac{1}{4}f(x)$

**8.** $f(x) = x^4 + x + 1,\ g(x) = f(-x) + 2$

**9.** Describe and correct the error in describing the transformation of the graph of $f(x) = x^4$ represented by the graph of $g(x) = 4x^4 + 3$.

> ✗ The graph of *g* is a vertical shrink by a factor of $\frac{1}{4}$, followed by a translation 3 units up of the graph of *f*.

**In Exercises 10 and 11, write a rule for *g* that represents the indicated transformations of the graph of *f*.**

**10.** $f(x) = x^3 - 3x^2 + 2;$ horizontal stretch by a factor of 3 and a translation 3 units up, followed by a reflection in the $x$-axis

**11.** $f(x) = 3x^5 - x^3 + 5x^2 + 1;$ reflection in the $y$-axis and a vertical shrink by a factor of $\frac{1}{2}$, followed by a translation 1 unit up

**12.** The volume $V$ (in cubic inches) of a rectangular box is given by $V = 2x^3 + 9$.

**a.** The function $W(x) = V\left(\frac{x}{12}\right)$ gives the volume (in cubic feet) of the box when $x$ is measured in inches. Write a rule for $W$. Find and interpret $W(6)$.

**b.** The function $Z(x) = W\left(\frac{x}{3}\right)$ gives the volume (in cubic yards) of the box when $x$ is measured in inches. Write a rule for $Z$.