Name____ Date__________

2.7 Analyzing Graphs of Polynomial Functions

For use with Exploration 2.7

Essential Question How many turning points can the graph of a polynomial function have?

1 EXPLORATION: Approximating Turning Points

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Match each polynomial function with its graph. Explain your reasoning. Then use a graphing calculator to approximate the coordinates of the turning points of the graph of the function. Round your answers to the nearest hundredth.

a. $f(x) = 2x^2 + 3x - 4$

b. $f(x) = x^2 + 3x + 2$

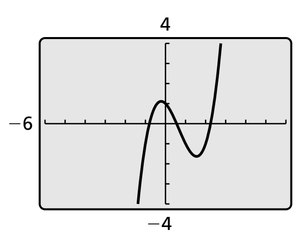
c. $f(x) = x^3 - 2x^2 - x + 1$

d. $f(x) = -x^3 + 5x - 2$

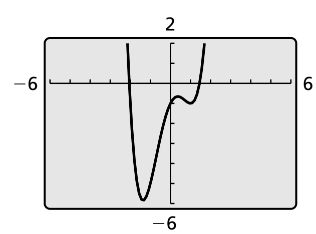
e. $f(x) = x^4 - 3x^2 + 2x - 1$

f. $f(x) = -2x^5 - x^2 + 5x + 3$

A.



B.



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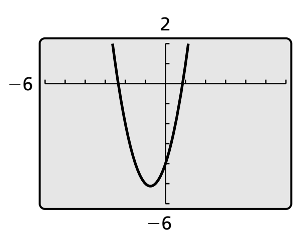

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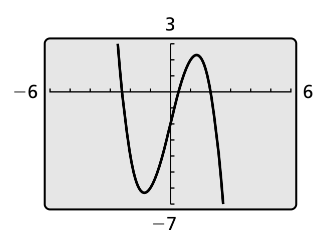
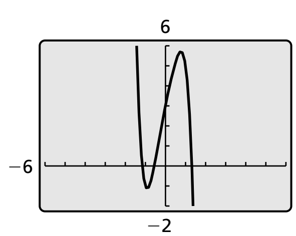
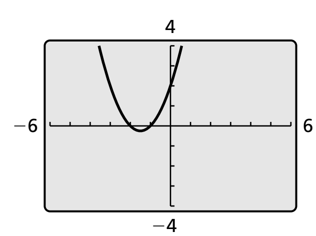


E.



F.



Communicate Your Answer

2. How many turning points can the graph of a polynomial function have?

3. Is it possible to sketch the graph of a cubic polynomial function that has *no* turning points? Justify your answer.

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2.7 Practice
For use after Lesson 2.7

Core Concepts

Zeros, Factors, Solutions, and Intercepts

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial function. The following statements are equivalent.

Zero: k is a zero of the polynomial function f.

Factor: $x - k$ is a factor of the polynomial $f(x)$.

Solution: k is a solution (or root) of the polynomial equation $f(x) = 0$.

***x*-Intercept:** If k is a real number, then k is an x-intercept of the graph of the polynomial function f. The graph of f passes through $(k, 0)$.

Notes:

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2.7 **Practice (continued)**

The Location Principle

If f is a polynomial function, and a and b are two real numbers such that $f(a) < 0$ and $f(b) > 0$, then f has at least one real zero between a and b.

Notes:

Turning Points of Polynomial Functions

1. The graph of every polynomial function of degree n has *at most* $n - 1$ turning points.

2. If a polynomial function has n distinct real zeros, then its graph has *exactly* $n - 1$ turning points.

Notes:

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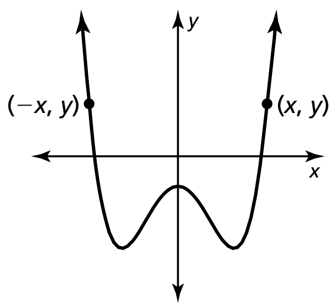
Even and Odd Functions

A function f is an **even function** when $f(-x) = f(x)$ for all x in its domain. The graph of an even function is *symmetric about the y-axis.*

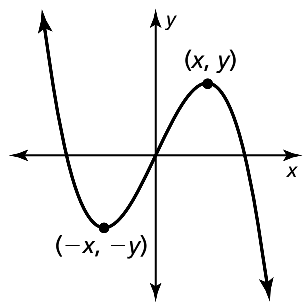
A function f is an **odd function** when $f(-x) = -f(x)$ for all x in its domain. The graph of an odd function is *symmetric about the origin*. One way to recognize a graph that is symmetric about the origin is that it looks the same after a $180°$ rotation about the origin.

Even Function



For an even function, if (x, y) is on the graph, then $(-x, y)$ is also on the graph.

Odd Function



For an odd function, if (x, y) is on the graph, then $(-x, -y)$ is also on the graph.

Notes:

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2.7 **Practice (continued)**

Worked-Out Examples

Example #1

Graph the function.

$g(x) = 4(x + 1)(x + 2)(x - 1)$

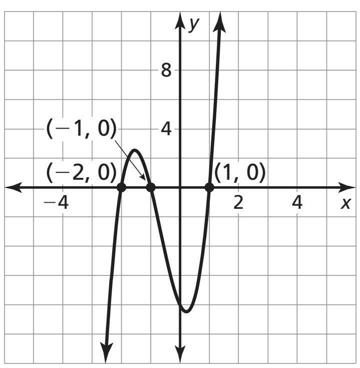
Step 1 Plot the x-intercepts. Because -1, -2, and 1 are zeros of g, plot $(-1, 0)$, $(-2, 0)$, and $(1, 0)$.

Step 2 Plot points between and beyond the x-intercepts.

x	-3	0	0.5	2
y	-32	-8	-7.5	48

Step 3 Determine end behavior. Because g has three factors of the form $x - k$ and a constant factor 4, it is a cubic function with a positive leading coefficient. So, $g(x) \rightarrow -\infty$ as $x \rightarrow -\infty$ and $g(x) \rightarrow +\infty$ as $x \rightarrow +\infty$.

Step 4 Draw the graph so that it passes through the plotted points and has the appropriate end behavior.

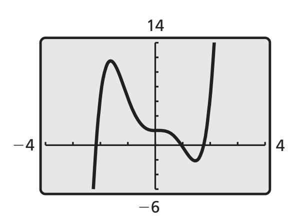


Example #2

Graph the function. Identify the x-intercepts and the points where the local maximums and the local minimums occur. Determine the intervals for which the function is increasing or decreasing.

$f(x) = x^5 - 4x^3 + x^2 + 2$

Use a graphing calculator to graph the function. The graph of f has three x-intercepts and two turning points. Use the graphing calculator's *zero*, *maximum*, and *minimum* features to approximate the coordinates of the points. The x-intercepts of the graph are $x \approx -2.16$, $x = 1$, and $x \approx 1.75$. The function has a local maximum at $(-1.63, 10.47)$ and a local minimum at $(1.46, -1.68)$. The function is increasing when $x < -1.63$ and $x > 1.46$ and decreasing when $-1.63 < x < 1.46$.



Practice A

In Exercises 1–6, graph the function. Identify the *x*-intercepts, and the points where the local maximums and local minimums occur. Determine the intervals for which the function is increasing or decreasing. Determine whether the function is *even*, *odd*, or *neither*.

1. $f(x) = 4x^3 - 12x^2 - x + 15$

2. $g(x) = 2x^4 + 5x^3 - 21x^2 - 10x$

3. $h(x) = x^3 - x^2 - 13x - 3$

4. $k(x) = x^3 - 2x$

5. $f(x) = x^4 - 29x^2 + 100$

6. $g(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$

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Practice B

In Exercises 1–4, graph the function.

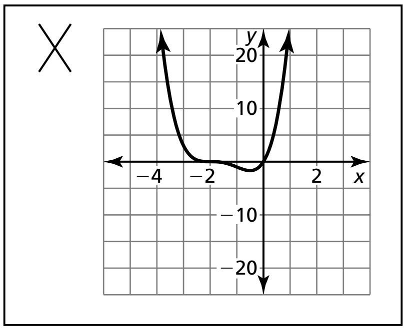
1. $f(x) = 4(x+3)^2(x-2)^2$

2. $g(x) = \frac{1}{2}(x-4)(x+3)(x-6)$

3. $h(x) = \frac{1}{5}(x-3)(x-4)(x+8)$

4. $f(x) = (x-2)(x^2+x+2)$

5. Describe and correct the error in using factors to graph $f(x) = x^2(x+2)^3$.



In Exercises 6–9, find all real zeros of the function.

6. $f(x) = 2x^3 - x^2 + 8x - 4$

7. $f(x) = 2x^3 + 7x^2 + x - 10$

8. $f(x) = 4x^3 - 3x^2 - 36x + 27$

9. $f(x) = 2x^3 + 3x^2 + 10x + 15$

In Exercises 10–13, graph the function. Identify the *x*-intercepts and the points where the local maximums and local minimums occur. Determine the intervals for which the function is increasing and decreasing.

10. $f(x) = 0.5x^3 - 3x^2 + 1.5$

11. $g(x) = 0.4x^3 - 3x$

12. $h(x) = x^5 - 3x^2 - 9x - 2$

13. $f(x) = x^4 - 3x^3 + 3x^2 + x - 2$

14. You are making a rectangular box out of a 12-inch by 8-inch piece of cardboard. The box will be formed by making the cuts shown in the diagram and folding up the sides. You want the box to have the greatest volume possible.

a. How long should you make the cuts?

b. What is the maximum volume?

c. What are the dimensions of the finished box?