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2.8 Modeling with Polynomial Functions
For use with Exploration 2.8

Essential Question How can you find a polynomial model for real-life data?

1 EXPLORATION: Modeling Real-Life Data

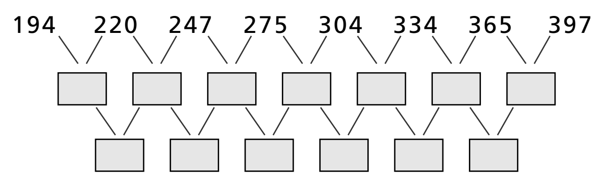
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. The distance a baseball travels after it is hit depends on the angle at which it was hit and the initial speed. The table shows the distances a baseball hit at an angle of 35° travels at various initial speeds.

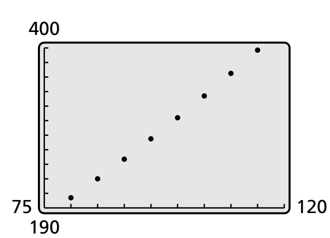
Initial speed, x (miles per hour)	80	85	90	95	100	105	110	115
Distance, y (feet)	194	220	247	275	304	334	365	397

a. Recall that when data have equally-spaced x-values, you can analyze patterns in the differences of the y-values to determine what type of function can be used to model the data. If the first differences are constant, then the set of data fits a linear model. If the second differences are constant, then the set of data fits a quadratic model.

Find the first and second differences of the data. Are the data linear or quadratic? Explain your reasoning.



b. Use a graphing calculator to draw a scatter plot of the data. Do the data appear linear or quadratic? Use the *regression* feature of the graphing calculator to find a linear or quadratic model that best fits the data.



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1 EXPLORATION: Modeling Real-Life Data (continued)

c. Use the model you found in part (b) to find the distance a baseball travels when it is hit at an angle of 35° and travels at an initial speed of 120 miles per hour.

d. According to the *Baseball Almanac,* "Any drive over 400 feet is noteworthy. A blow of 450 feet shows exceptional power, as the majority of major league players are unable to hit a ball that far. Anything in the 500-foot range is genuinely historic." Estimate the initial speed of a baseball that travels a distance of 500 feet.

Communicate Your Answer

2. How can you find a polynomial model for real-life data?

3. How well does the model you found in Exploration 1(b) fit the data? Do you think the model is valid for any initial speed? Explain your reasoning.

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2.8 Practice
For use after Lesson 2.8

Core Concepts

Properties of Finite Differences

1. If a polynomial function $y = f(x)$ has degree n, then the nth differences of function values for equally-spaced x-values are nonzero and constant.

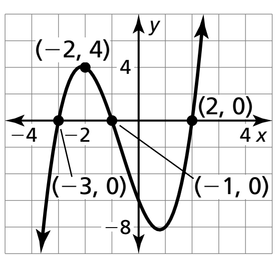
2. Conversely, if the nth differences of equally-spaced data are nonzero and constant, then the data can be represented by a polynomial function of degree n.

Notes:

Worked-Out Examples

Example #1

Write a cubic function whose graph is shown.



Step 1 Use the three x-intercepts to write the function in factored form.

$f(x) = a(x + 3)(x + 1)(x - 2)$

Step 2 Find the value of a by substituting the coordinates of the point $(-2, 4)$.

$4 = a(-2 + 3)(-2 + 1)(-2 - 2)$

$4 = 4a$

$1 = a$

The function is $f(x) = (x + 3)(x + 1)(x - 2)$.

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2.8 Practice (continued)

Example #2

Use finite differences to determine the degree of the polynomial function that fits the data. Then use technology to find the polynomial function.

x	−6	−3	0	3	6	9
$f(x)$	−2	15	−4	49	282	803

Step 1 Write the function values. Find the first differences by subtracting consecutive values. Then find the second differences by subtracting the consecutive first differences. Continue until you obtain differences that are nonzero and constant.

$f(-6)$ $f(-3)$ $f(0)$ $f(3)$ $f(6)$ $f(9)$
−2 15 −4 49 282 803
17 −19 53 233 521
−36 72 180 288
108 108 108

Because the third differences are nonzero and constant, you can model the data exactly with a cubic function.

Step 2 Enter the data into a graphing calculator and use the *cubic regression* feature to obtain a cubic model.

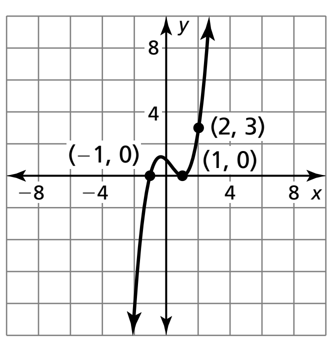
Because $\frac{2}{3} \approx 0.666667$ and $\frac{1}{3} \approx 0.333333$, a polynomial function that fits the data exactly is

$f(x) = \frac{2}{3}x^3 + 4x^2 - \frac{1}{3}x - 4.$

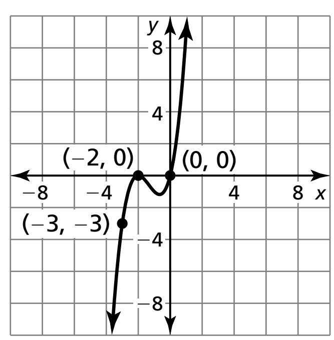
Practice A

In Exercises 1–4, write a cubic function whose graph is shown.

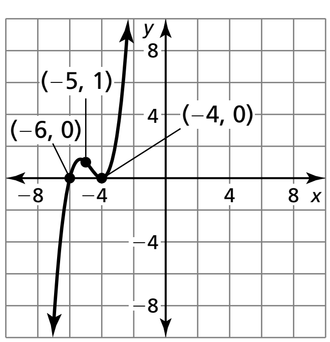
1.



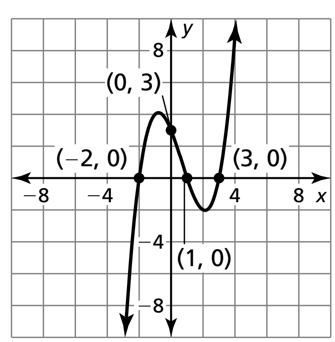
2.



3.



4.



2.8 Practice (continued)

In Exercises 5–8, use finite differences to determine the degree of the polynomial function that fits the data. Then use technology to find the polynomial function.

5.

x	−2	−1	0	1	2	3
$f(x)$	−14	−6.5	0	5.5	10	13.5

6.

x	−2	−1	0	1	2
$f(x)$	30	4	0	0	−14

7. $(0, 0), (2, 0), (4, 40), (6, 168), (8, 432), (10, 880)$

8. $(0, 10), (1, 10), (2, 18), (3, 64), (4, 202), (5, 510)$

9. The table shows the population y of bacteria after x hours. Find a polynomial model for the data for the first 4.5 hours. Use the model to estimate the population level of the bacteria (in thousands) after 1 day.

Number of Hours, *x*	0.5	1	2.5	3	4	4.5
Number of Bacteria, *y*	5.125	6	20.625	32	69	96.125

10. The table shows the value y (in thousands of dollars) of a signed autograph of a MVP football player, where x represents the number of years since 2000. Find a polynomial model for the data for the first 5 years. Use the model to estimate the year that the autograph will be valued at $5,000,000?

Number of Years, *x*	1	2	3	4	5
Value of autograph, *y*	6	34	162	510	1246

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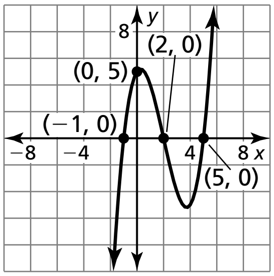
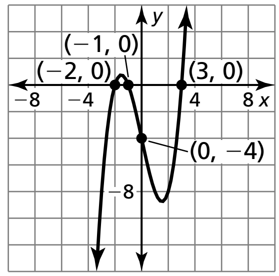
Practice B

In Exercises 1 and 2, write a cubic function whose graph is shown.

1.



2.



In Exercises 3–5, use finite differences to determine the degree of the polynomial function that fits the data. Then use technology to find the polynomial function.

3.

x	1	2	3	4	5	6
***f*(*x*)**	−10	−14	−13	−7	4	20

4.

x	1	2	3	4	5	6	7	8
***f*(*x*)**	24	12	6	9	24	54	102	171

5.

x	1	2	3	4	5	6	7
***f*(*x*)**	20	4	0	4	16	40	84

6. The data in the table show the wave height (in inches) over a 7-second period.

x	1	2	3	4	5	6	7
***f*(*x*)**	0.5	3	6.5	12	20.5	33	50.5

a. Find a polynomial model for the data.

b. Does this model seem reasonable for the next 7-second interval? Explain.