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2.9 Performing Function Operations
For use with Exploration 2.9

Essential Question How can you use the graphs of two functions to sketch the graph of an arithmetic combination of the two functions?

Just as two real numbers can be combined by the operations of addition, subtraction, multiplication, and division to form other real numbers, two functions can be combined to form other functions. For example, the functions $f(x) = 2x - 3$ and $g(x) = x^2 - 1$ can be combined to form the sum, difference, product, or quotient of *f* and *g*.

$f(x) + g(x) = (2x - 3) + (x^2 - 1) = x^2 + 2x - 4$ sum

$f(x) - g(x) = (2x - 3) - (x^2 - 1) = -x^2 + 2x - 2$ difference

$f(x) \bullet g(x) = (2x - 3)(x^2 - 1) = 2x^3 - 3x^2 - 2x + 3$ product

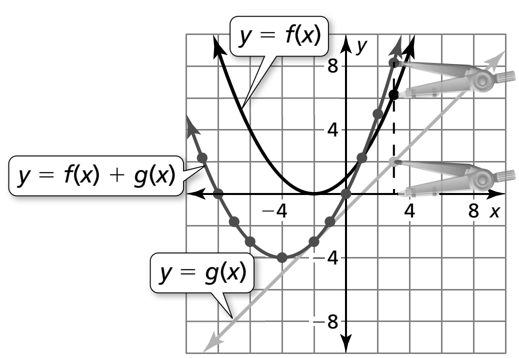
$\dfrac{f(x)}{g(x)} = \dfrac{2x - 3}{x^2 - 1}$ quotient

1 EXPLORATION: Graphing the Sum of Two Functions

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use the graphs of *f* and *g* to sketch the graph of $f + g$. Explain your steps.

Sample Choose a point on the graph of *g*. Use a compass or a ruler to measure its distance above or below the *x*-axis. If above, add the distance to the *y*-coordinate of the point with the same *x*-coordinate of the graph of *f*. If below, subtract the distance. Plot the new point. Repeat this process for several points. Finally, draw a smooth curve through the new points to obtain the graph of $f + g$.



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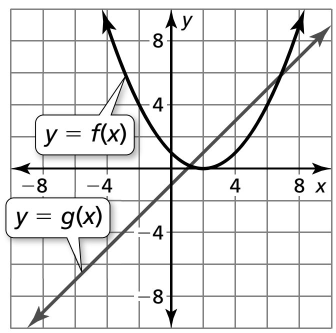
1 EXPLORATION: Graphing the Sum of Two Functions (continued)

a.



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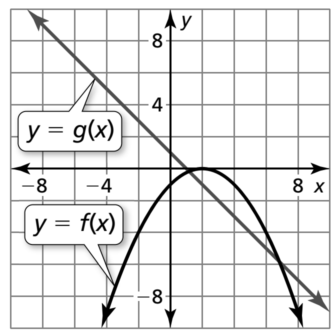
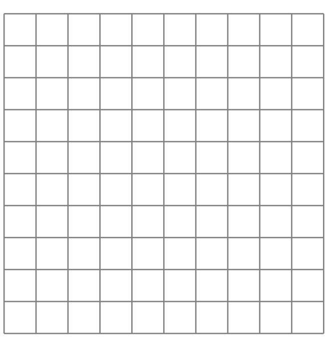
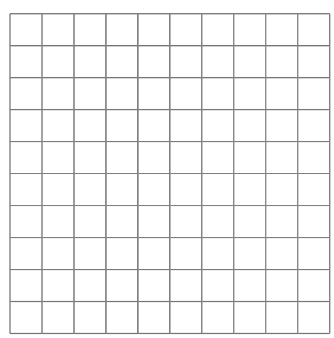


Communicate Your Answer

2. How can you use the graphs of two functions to sketch the graph of an arithmetic combination of the two functions?

3. Check your answers in Exploration 1 by writing equations for *f* and *g*, adding the functions, and graphing the sum.





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2.9 Practice

For use after Lesson 2.9

Core Concepts

Operations on Functions

Let f and g be any two functions. A new function can be defined by performing any of the four basic operations on f and g.

Operation	Definition	Example: $f(x) = 5x$, $g(x) = x + 2$
Addition	$(f + g)(x) = f(x) + g(x)$	$(f + g)(x) = 5x + (x + 2) = 6x + 2$
Subtraction	$(f - g)(x) = f(x) - g(x)$	$(f - g)(x) = 5x - (x + 2) = 4x - 2$
Multiplication	$(fg)(x) = f(x) \bullet g(x)$	$(fg)(x) = 5x(x + 2) = 5x^2 + 10x$
Division	$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$	$\left(\dfrac{f}{g}\right)(x) = \dfrac{5x}{x + 2}$

The domains of the sum, difference, product, and quotient functions consist of the x-values that are in the domains of both f and g. Additionally, the domain of the quotient does not include x-values for which $g(x) = 0$.

Notes:

Worked-Out Examples

Example #1

Find $(f + g)(x)$ and $(f - g)(x)$ and state the domain of each. Then evaluate $f + g$ and $f - g$ for the given value of x.

$f(x) = 11x + 2x^2$, $g(x) = -7x - 3x^2 + 4$; $x = 2$

$$\begin{aligned}(f + g)(x) &= f(x) + g(x)\\ &= (11x + 2x^2) + (-7x - 3x^2 + 4)\\ &= -x^2 + 4x + 4\end{aligned}$$

The functions f and g each have the same domain: all real numbers. So, the domain of $f + g$ is all real numbers. When $x = 2$, the value of the sum is

$(f + g)(2) = -(2)^2 + 4(2) + 4 = 8.$

$$\begin{aligned}(f - g)(x) &= f(x) - g(x)\\ &= (11x + 2x^2) - (-7x - 3x^2 + 4)\\ &= 5x^2 + 18x - 4\end{aligned}$$

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The functions f and g each have the same domain: all real numbers. So, the domain of $f - g$ is all real numbers. When $x = 2$, the value of the difference is

$(f - g)(2) = 5(2)^2 + 18(2) - 4 = 52.$

Example #2

Find $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ and state the domain of each. Then evaluate fg and $\frac{f}{g}$ for the given value of x.

$f(x) = x^4,\ g(x) = 3\sqrt{x};\ \ x = 4$

$(fg)(x) = (x^4)(3\sqrt{x}) = 3x^{9/2}$

The domain of f is all real numbers and the domain of g is $x \geq 0$. So, the domain of fg is $x \geq 0$. When $x = 4$, the value of the product is $(fg)(4) = 3(4)^{9/2} = 3(512) = 1536$.

$\left(\frac{f}{g}\right)(x) = \frac{x^4}{3\sqrt{x}} = \frac{x^{7/2}}{3}$

The domain of f is all real numbers and the domain of g is $x \geq 0$. So, the domain of $\frac{f}{g}$ is $x > 0$. When $x = 4$, the value of the quotient is $\left(\frac{f}{g}\right)(4) = \frac{4^{7/2}}{3} = \frac{128}{3}$.

Practice A

In Exercises 1–4, find $(f + g)(x)$ and $(f - g)(x)$ and state the domain of each. Then evaluate $f + g$ and $f - g$ for the given value of x.

1. $f(x) = -\frac{1}{2}\sqrt[3]{x},\ g(x) = \frac{9}{2}\sqrt[3]{x};\ x = -1000$

2. $f(x) = -x^2 - 3x + 8,\ g(x) = 6x - 3x^2;\ x = -1$

3. $f(x) = 4x^3 + 12,\ g(x) = 2x^2 - 3x^3 + 9;\ x = 2$

4. $f(x) = 5\sqrt[4]{x} + 1,\ g(x) = -3\sqrt[4]{x} - 2;\ x = 1$

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2.9 Practice (continued)

In Exercises 5–8, find $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ and state the domain of each. Then evaluate fg and $\frac{f}{g}$ for the given value of x.

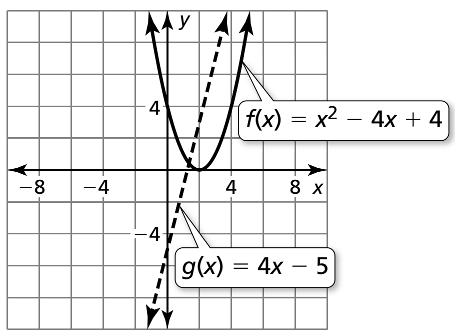
5. $f(x) = -x^3, g(x) = 2\sqrt[3]{x}; x = -64$

6. $f(x) = 12x, g(x) = 11x^{1/2}; x = 4$

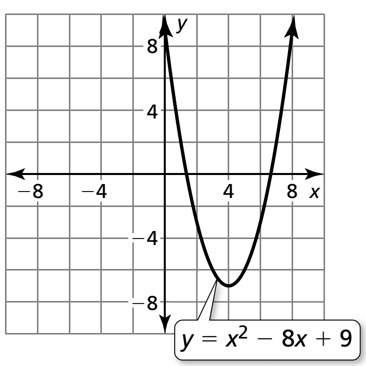
7. $f(x) = 0.25x^{1/3}, g(x) = -4x^{3/2}; x = 1$

8. $f(x) = 36x^{7/4}, g(x) = 4x^{1/2}; x = 16$

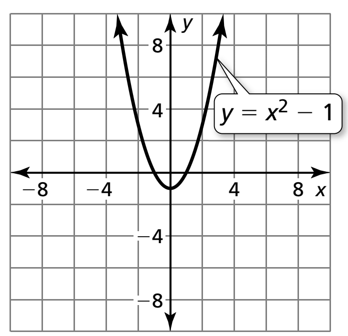
9. The graphs of the functions $f(x) = x^2 - 4x + 4$ and $g(x) = 4x - 5$ are shown. Which graph represents the function $f + g$? the function $f - g$? Explain your reasoning.



A.



B.



10. The variable x represents the number of pages of a textbook to be printed. The cost C to print the textbook can be modeled by the equation $C(x) = 0.2x^2 + 10$. The selling price P of the textbook can be modeled by the equation $P(x) = 0.05x^2 + 20$.

a. Find $(P - C)(x)$.

b. Explain what $(P - C)(x)$ represents.

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Practice B

In Exercises 1 and 2, find $(f + g)(x)$ and $(f - g)(x)$ and state the domain of each. Then evaluate $f + g$ and $f - g$ for the given value of x.

1. $f(x) = \sqrt[3]{4x}; g(x) = -9\sqrt[3]{4x}; x = -2$

2. $f(x) = 3x - 5x^2 - x^3; g(x) = 6x^2 - 4x; x = -1$

In Exercises 3–5, find $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ and state the domain of each. Then evaluate fg and $\frac{f}{g}$ for the given value of x.

3. $f(x) = 3x^3; g(x) = \sqrt[3]{x^2}; x = -8$

4. $f(x) = 3x^2; g(x) = 5x^{1/4}; x = 16$

5. $f(x) = 10x^{5/6}; g(x) = 2x^{1/3}; x = 64$

In Exercises 6 and 7, use a graphing calculator to evaluate $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\left(\frac{f}{g}\right)(x)$ when $x = 5$. Round your answers to two decimal places.

6. $f(x) = -3x^{1/3}; g(x) = 4x^{1/2}$

7. $f(x) = 6x^{3/4}; g(x) = 3x^{1/2}$

8. Describe and correct the error in stating the domain.

> ✗ $f(x) = 4x^{7/3}$ and $g(x) = 2x^{2/3}$
>
> The domain of $\left(\frac{f}{g}\right)(x)$ is all real numbers.

9. The table shows the outputs of the two functions f and g. Use the table to evaluate $(f + g)(5)$, $(f - g)(0)$, $(fg)(3)$, and $\left(\frac{f}{g}\right)(2)$.

x	0	1	2	3	4	5
f(x)	18	13	8	3	–2	–7
g(x)	64	32	16	8	4	2