

2.10**Inverse of a Function**

For use with Exploration 2.10

Essential Question How can you sketch the graph of the inverse of a function?

1 EXPLORATION: Graphing Functions and Their Inverses

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Each pair of functions are *inverses* of each other. Use a graphing calculator to graph f and g in the same viewing window. What do you notice about the graphs?

a. $f(x) = 4x + 3$

$$g(x) = \frac{x - 3}{4}$$

b. $f(x) = x^3 + 1$

$$g(x) = \sqrt[3]{x - 1}$$

c. $f(x) = \sqrt{x - 3}$

$$g(x) = x^2 + 3, x \geq 0$$

d. $f(x) = -\sqrt{x} + 2$

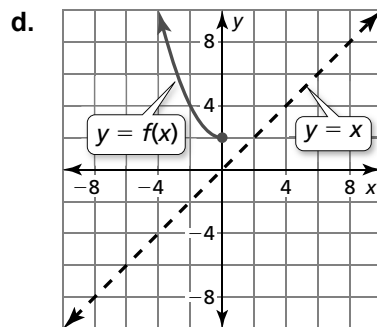
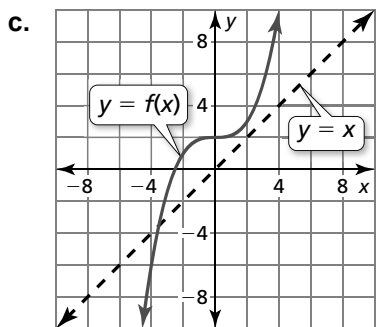
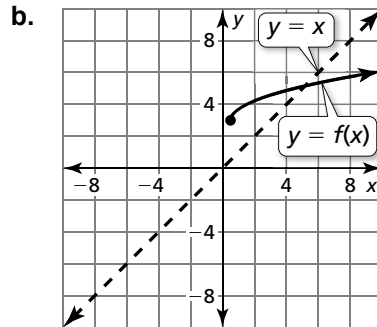
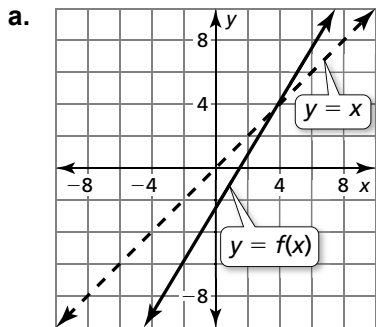
$$g(x) = (2 - x)^2, x \leq 2$$

2.10 Inverse of a Function (continued)

2 EXPLORATION: Sketching Graphs of Inverse Functions

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use the graph of f to sketch the graph of g , the inverse function of f , on the same set of coordinate axes. Explain your reasoning.



Communicate Your Answer

- How can you sketch the graph of the inverse of a function?
- In Exploration 1, what do you notice about the relationship between the equations of f and g ? Use your answer to find g , the inverse function of

$$f(x) = 2x - 3.$$

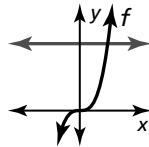
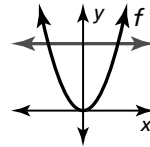
Use a graph to check your answer.

2.10**Practice**

For use after Lesson 2.10

Core Concepts**Horizontal Line Test**

The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f more than once.

Inverse is a function**Inverse is not a function****Notes:****Worked-Out Examples****Example #1**

Solve $y = f(x)$ for x . Then find the input(s) when the output is -3 .

$$f(x) = -\frac{2}{3}x + 1$$

$$y = -\frac{2}{3}x + 1$$

$$y - 1 = -\frac{2}{3}x$$

$$-\frac{3}{2}(y - 1) = x$$

$$-\frac{3y - 3}{2} = x$$

Find the input when $y = -3$.

$$x = -\frac{3}{2}(-3 - 1)$$

$$= -\frac{3}{2}(-4)$$

$$= 6$$

So, the input is 6 when the output is -3 .

2.10 Practice (continued)

Example #2

Find the inverse of the function. Then graph the function and its inverse.

$$f(x) = 6x - 3$$

Method 1 Use inverse operations in reverse order.

$$f(x) = 6x - 3$$

To find the inverse, apply inverse operations in the reverse order.

$$g(x) = \frac{1}{6}(x + 3)$$

The inverse of f is $g(x) = \frac{1}{6}(x + 3)$ or

$$g(x) = \frac{x + 3}{6}.$$

Method 2 Set y equal to $f(x)$. Switch the roles of x and y and solve for y .

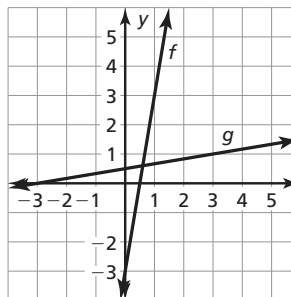
$$y = 6x - 3$$

$$x = 6y - 3$$

$$x + 3 = 6y$$

$$\frac{x + 3}{6} = y$$

The inverse of f is $g(x) = \frac{x + 3}{6}$.



Practice A

In Exercises 1–3, solve $y = f(x)$ for x . Then find the input(s) when the output is -6 .

1. $f(x) = 2x - 1$

2. $f(x) = 1 - x^2$

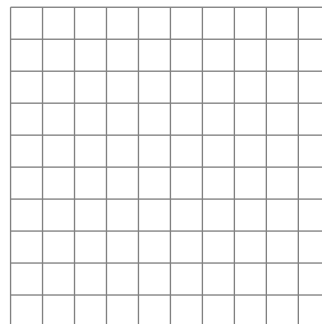
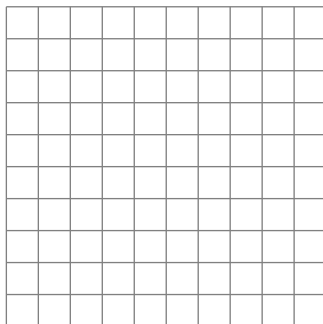
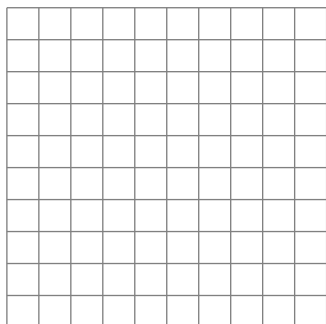
3. $f(x) = (x - 1)^3 + 2$

In Exercises 4–6, find the inverse of the function. Then graph the function and its inverse.

4. $f(x) = 10x$

5. $f(x) = -\frac{1}{5}x - 7$

6. $f(x) = \frac{3}{4}x + \frac{5}{8}$



2.10 Practice (continued)

In Exercises 7 and 8, determine whether each pair of functions f and g are inverses. Explain your reasoning.

7.

x	-4	-3	-2	-1	0	1
$f(x)$	17	13	9	5	1	-3

x	17	13	9	5	1	-3
$g(x)$	-4	-3	-2	-1	0	1

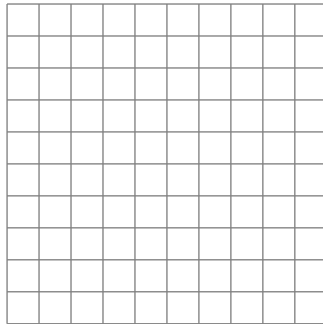
8.

x	1	2	3	4	5	6
$f(x)$	-1	-2	-4	-5	-8	-10

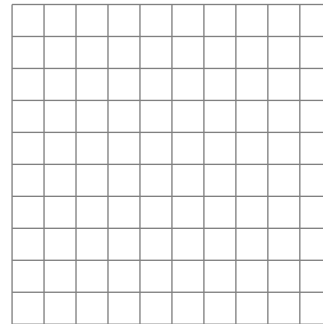
x	-1	-0.5	-0.25	-0.2	-0.125	-0.1
$g(x)$	1	2	3	4	5	6

In Exercises 9 and 10, find the inverse of the function. Then graph the function and its inverse.

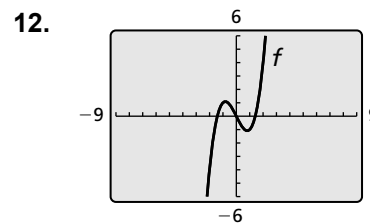
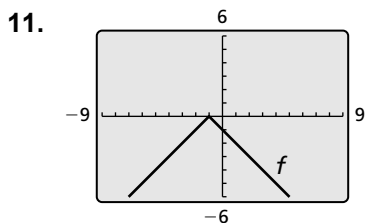
9. $f(x) = (x + 2)^3$



10. $f(x) = \frac{1}{3}x^4, x \geq 0$



In Exercises 11 and 12, use the graph to determine whether the inverse of f is a function. Explain your reasoning.



In Exercises 13 and 14, determine whether the functions are inverse functions.

13. $f(x) = \frac{4x}{5} - 1, g(x) = \frac{5x + 1}{4}$

14. $f(x) = -(x - 2)^5 + 6, g(x) = 2 + (6 - x)^{1/5}$

Practice B

In Exercises 1–3, solve $y = f(x)$ for x . Then find the input(s) when the output is -3 .

1. $f(x) = -\frac{4}{3}x + 2$

2. $f(x) = 25x^4$

3. $f(x) = (x - 3)^2 - 4$

In Exercises 4–6, find the inverse of the function. Then graph the function and its inverse.

4. $f(x) = -3x + 4$

5. $f(x) = -\frac{1}{3}x + 1$

6. $f(x) = \frac{2}{5}x - \frac{1}{5}$

7. Describe and correct the error in finding the inverse function.

\times	$f(x) = 3x - 8$ $y = 3x - 8$ $x = 3y - 8$ $g(x) = 3x - 8$
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In Exercises 8–10, find the inverse function. Then graph the function and its inverse.

8. $f(x) = -9x^2, x \leq 0$

9. $f(x) = (x - 1)^3$

10. $f(x) = x^6, x \leq 0$

11. Find the inverse of the function $f(x) = 8x^3$ by switching the roles of x and y and solving for y . Then find the inverse of the function f by using inverse operations in the reverse order. Which method do you prefer? Explain.

In Exercises 12–15, determine whether the functions are inverse functions.

12. $f(x) = 6x + 1; g(x) = 6x - 1$

13. $f(x) = \frac{\sqrt[3]{x-6}}{2}; g(x) = 8x^3 + 6$

14. $f(x) = \frac{5-x}{2}; g(x) = 5 - 2x$

15. $f(x) = 4x^2 + 3; g(x) = -\frac{x-3}{4}$

16. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$, where r is the radius.

a. Find the inverse function. Describe what it represents.

b. Find the radius of a sphere with a volume of 146 cubic meters.