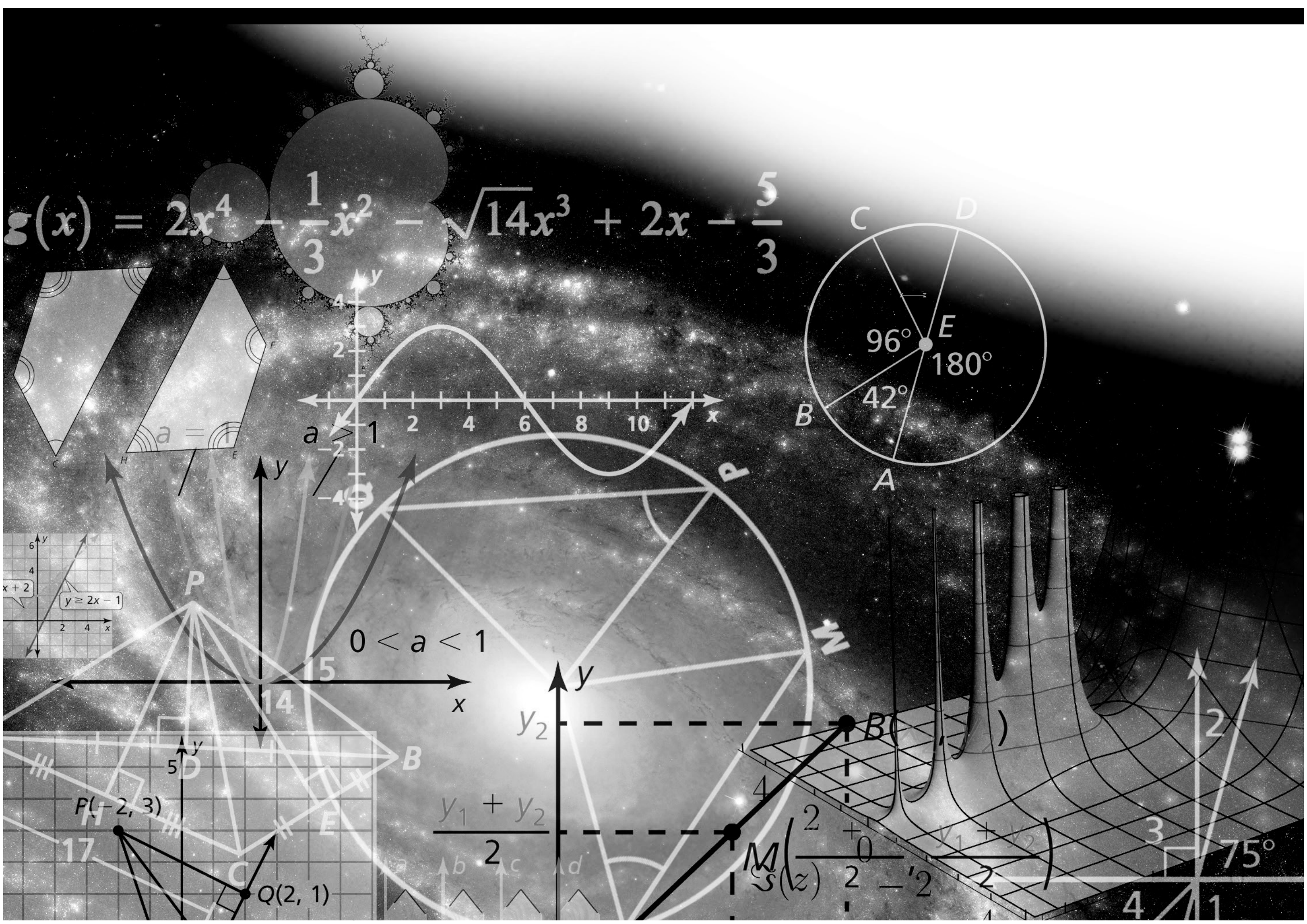
CHAPTER 3

Exponential and Logarithmic Functions



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Chapter 3 Maintaining Mathematical Proficiency

Evaluate the expression.

1. $-4 \bullet 5^3$ **2.** $(-3)^4$ **3.** $-\left(\frac{7}{8}\right)^2$ **4.** $\left(\frac{3}{10}\right)^3$

Tell whether the function represents *exponential growth* or *exponential decay*. Then graph the function.

5. $f(x) = 0.5^x$ **6.** $y = 4^x$

7. $g(x) = 2.6^x$ **8.** $h(x) = 0.25^x$

9. Is the expression "the sum of the square of x and the square of the opposite of x" equivalent to 0 or $2x^2$? Explain your reasoning.

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3.1 Exponential Functions
For use with Exploration 3.1

Essential Question What are some of the characteristics of the graph of an exponential function?

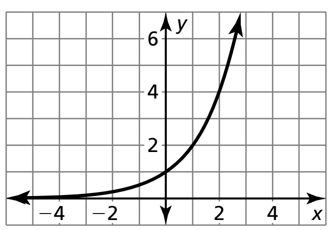
1 EXPLORATION: Identifying Graphs of Exponential Functions

Work with a partner. Match each exponential function with its graph. Use a table of values to sketch the graph of the function, if necessary.

a. $f(x) = 2^x$ **b.** $f(x) = 3^x$ **c.** $f(x) = 4^x$

d. $f(x) = \left(\frac{1}{2}\right)^x$ **e.** $f(x) = \left(\frac{1}{3}\right)^x$ **f.** $f(x) = \left(\frac{1}{4}\right)^x$

A.



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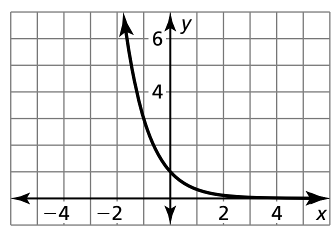

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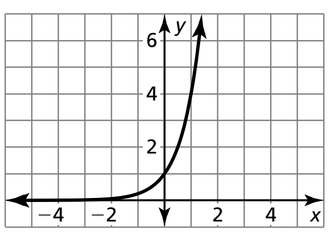

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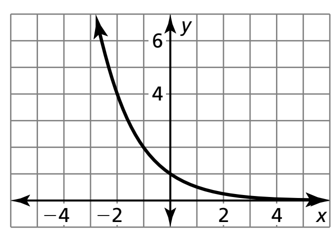
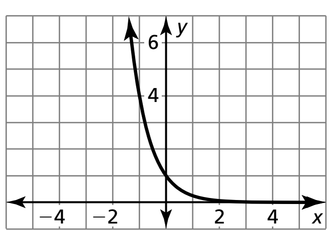
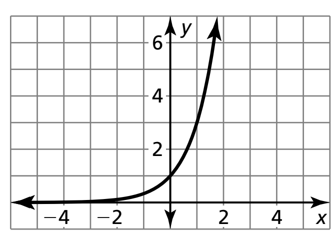


E.



F.



2 EXPLORATION: Characteristics of Graphs of Exponential Functions

Work with a partner. Use the graphs in Exploration 1 to determine the domain, range, and y-intercept of the graph of $f(x) = b^x$, where b is a positive real number other than 1. Explain your reasoning.

Communicate Your Answer

3. What are some of the characteristics of the graph of an exponential function?

4. In Exploration 2, is it possible for the graph of $f(x) = b^x$ to have an x-intercept? Explain your reasoning.

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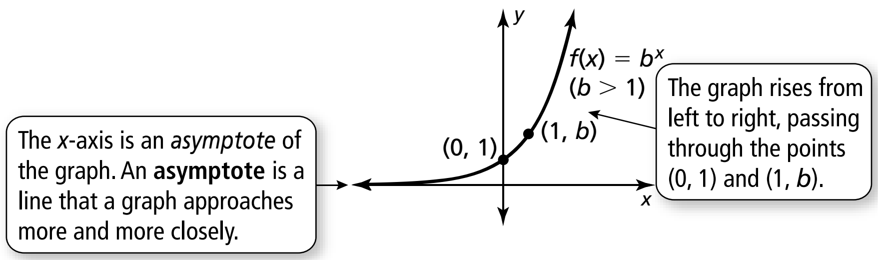
3.1 Practice
For use after Lesson 3.1

Core Concepts

Parent Function for Exponential Growth Functions

The function $f(x) = b^x$, where $b > 1$, is the parent function for the family of exponential growth functions with base b. The graph shows the general shape of an exponential growth function.

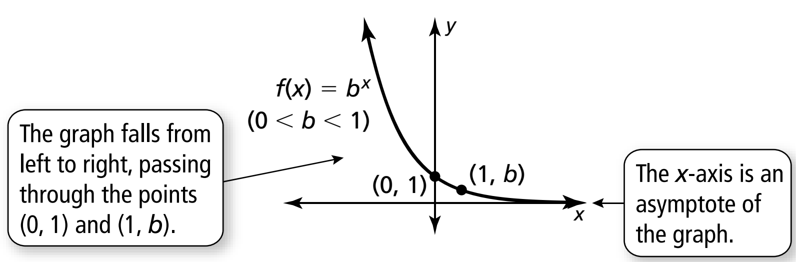


The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

Notes:

Parent Function for Exponential Decay Functions

The function $f(x) = b^x$, where $0 < b < 1$, is the parent function for the family of exponential decay functions with base b. The graph shows the general shape of an exponential decay function.



The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

Notes:

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3.1 Practice (continued)

Writing Recursive Rules for Exponential Functions

An exponential function of the form $f(x) = ab^x$ is written using a recursive rule as follows.

Recursive Rule $f(0) = a, f(n) = r \bullet f(n-1)$, where $a \neq 0$, r is the common ratio, and n is a natural number.

Example $y = 6(3)^x$ can be written as $f(0) = 6, f(n) = 3 \bullet f(n-1)$

initial value

common ratio

Notes:

Worked-Out Examples

Example #1

Evaluate the expression for (a) *x* = – 2 and (b) *x* = 3.

$6 \bullet 2^x$

a. $6 \bullet 2^{-2} = 6 \bullet \frac{1}{4} = \frac{3}{2}$

b. $6 \bullet 2^3 = 6 \bullet 8 = 48$

Example #2

Determine whether the function represents exponential growth or exponential decay. Then graph the function.

$y = \left(\frac{1}{8}\right)^x$

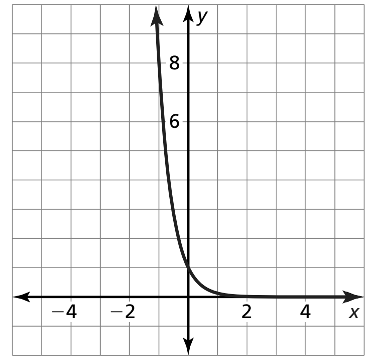
Step 1 Identify the value of the base. The base, $\frac{1}{8}$, is greater than 0 and less than 1, so the function represents exponential decay.

Step 2 Make a table of values.

x	-1	0	1	2
y	8	1	$\frac{1}{8}$	$\frac{1}{64}$

Step 3 Plot the points from the table.

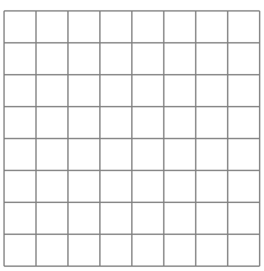
Step 4 Draw, from right to left, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the left.



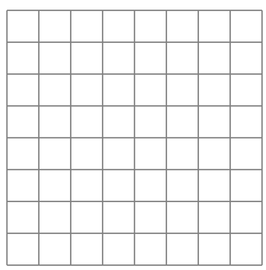
Practice A

In Exercises 1–4, determine whether the function represents *exponential growth* or *exponential decay.* Then graph the function.

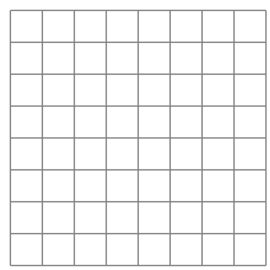
1. $y = \left(\frac{1}{12}\right)^x$



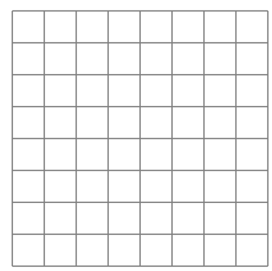
2. $y = (1.5)^x$



3. $y = \left(\frac{7}{2}\right)^x$



4. $y = (0.8)^x$



5. The number of bacteria y (in thousands) in a culture can be approximated by the model $y = 100(1.99)^t$, where t is the number of hours the culture is incubated.

a. Tell whether the model represents exponential growth or exponential decay.

b. Identify the hourly percent increase or decrease in the number of bacteria.

c. Estimate when the number of bacteria will be 1,000,000.

In Exercises 6 and 7, write a recursive rule for the exponential function.

6. $y = 2(6)^x$

7. $f(t) = 11(0.3)^t$

In Exercises 8 and 9, rewrite the function to determine whether it represents *exponential growth* or *exponential decay*. Then identify the percent rate of change.

8. $y = 3(1.25)^{t+4}$

9. $f(t) = (0.44)^{7t}$

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Practice B

In Exercises 1–3, evaluate the expression for (a) $x = -2$ and (b) $x = 3$.

1. 5^x **2.** $10 \bullet 2^x$ **3.** $3^x - 3$

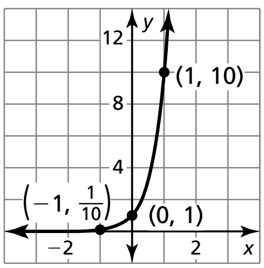
In Exercises 4–9, tell whether the function represents *exponential growth* or *exponential decay*. Then graph the function.

4. $y = 8^x$ **5.** $y = \left(\frac{5}{3}\right)^x$ **6.** $y = \left(\frac{2}{3}\right)^x$

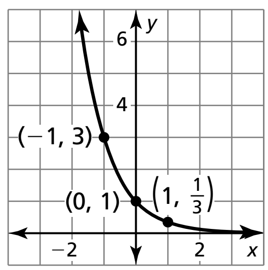
7. $y = (2.5)^x$ **8.** $y = (0.4)^x$ **9.** $y = (0.1)^x$

In Exercises 10 and 11, use the graph of $f(x) = b^x$ to identify the value of the base b.

10.



11.



12. The value of a truck y (in dollars) can be approximated by the model $y = 54{,}000(0.80)^t$, where t is the number of years since the truck was new.

a. Tell whether the model represents exponential growth or exponential decay.

b. Identify the annual percent increase or decrease in the value of the truck.

c. What was the original value of the truck?

d. Estimate when the value of the truck will be \$30,000.

In Exercises 13–15, write a recursive rule for the exponential function.

13. $y = 18(5)^x$ **14.** $y = 0.2(10)^x$ **15.** $y = 6\left(\frac{1}{3}\right)^x$

In Exercises 16–18, rewrite the function in the form $y = a(1 + r)^t$ or $y = a(1 - r)^t$. Then state the growth or decay rate.

16. $y = a(0.75)^{t/6}$ **17.** $y = a\left(\frac{4}{3}\right)^{t/18}$ **18.** $y = a\left(\frac{1}{4}\right)^{4t}$