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# 3.2 The Natural Base *e*

**For use with Exploration 3.2**

## Essential Question What is the natural base *e*?

So far in your study of mathematics, you have worked with special numbers such as $\pi$ and $i$. Another special number is called the *natural base* and is denoted by $e$. The natural base $e$ is irrational, so you cannot find its exact value.

## 1 EXPLORATION: Approximating the Natural Base *e*

**Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.**

**Work with a partner.** One way to approximate the natural base $e$ is to approximate the sum

$$1 + \frac{1}{1} + \frac{1}{1 \bullet 2} + \frac{1}{1 \bullet 2 \bullet 3} + \frac{1}{1 \bullet 2 \bullet 3 \bullet 4} + \cdots.$$

Use a spreadsheet or a graphing calculator to approximate this sum. Explain the steps you used. How many decimal places did you use in your approximation?

## 2 EXPLORATION: Approximating the Natural Base *e*

**Work with a partner.** Another way to approximate the natural base $e$ is to consider the expression

$$\left(1 + \frac{1}{x}\right)^x.$$

As $x$ increases, the value of this expression approaches the value of $e$. Complete the table. Then use the results in the table to approximate $e$. Compare this approximation to the one you obtained in Exploration 1.

| $x$ | $10^1$ | $10^2$ | $10^3$ | $10^4$ | $10^5$ | $10^6$ |
|---|---|---|---|---|---|---|
| $\left(1 + \frac{1}{x}\right)^x$ | | | | | | |

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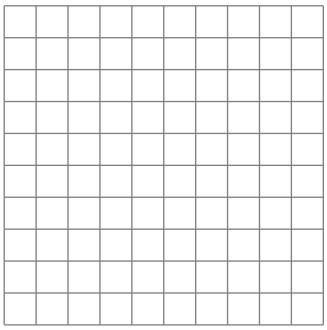
## 3 EXPLORATION: Graphing a Natural Base Function

**Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.**

**Work with a partner.** Use your approximate value of $e$ in Exploration 1 or 2 to complete the table. Then sketch the graph of the *natural base exponential function* $y = e^x$. You can use a graphing calculator and the $e^x$ key to check your graph. What are the domain and range of $y = e^x$? Justify your answers.

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y = e^x$ | | | | | |

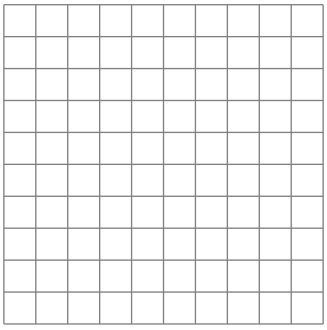


## Communicate Your Answer

**4.** What is the natural base $e$?

**5.** Repeat Exploration 3 for the natural base exponential function $y = e^{-x}$. Then compare the graph of $y = e^x$ to the graph of $y = e^{-x}$.

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y = e^{-x}$ | | | | | |



**6.** The natural base $e$ is used in a wide variety of real-life applications. Use the Internet or some other reference to research some of the real-life applications of $e$.

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# 3.2 Practice

**For use after Lesson 3.2**

## Core Concepts

### The Natural Base *e*

The natural base $e$ is irrational. It is defined as follows:

As $x$ approaches $+\infty$, $\left(1 + \frac{1}{x}\right)^x$ approaches $e \approx 2.71828182846$.

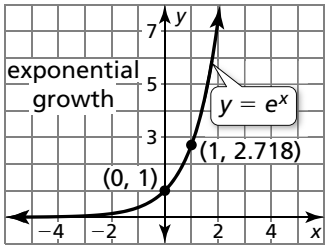
**Notes:**

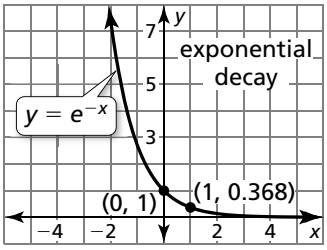
### Natural Base Functions

A function of the form $y = ae^{rx}$ is called a *natural base exponential function*.

- When $a > 0$ and $r > 0$, the function is an exponential growth function.
- When $a > 0$ and $r < 0$, the function is an exponential decay function.

The graphs of the basic functions $y = e^x$ and $y = e^{-x}$ are shown.




**Notes:**

### Continuously Compounded Interest

When interest is compounded *continuously*, the amount $A$ in an account after $t$ years is given by the formula

$$A = Pe^{rt}$$

where $P$ is the principal and $r$ is the annual interest rate expressed as a decimal.

**Notes:**

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**3.2 Practice (continued)**

## Worked-Out Examples

### Example #1

**Simplify the expression.**

$$\frac{27e^7}{3e^4} = 9e^{7-4}$$
$$= 9e^3$$

### Example #2

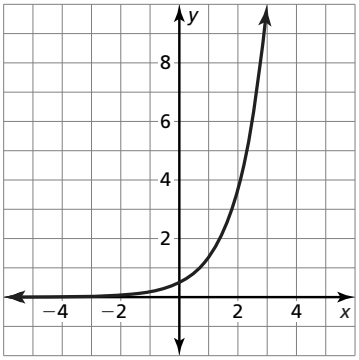
**Tell whether the function represents exponential growth exponential decay. Then graph the function.**

$y = 0.5e^x$

Because $a = 0.5$ is positive and $r = 1$ is positive, the function is an exponential growth function.

Use a table to graph the function.

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 0.07 | 0.18 | 0.5 | 1.36 | 3.69 |



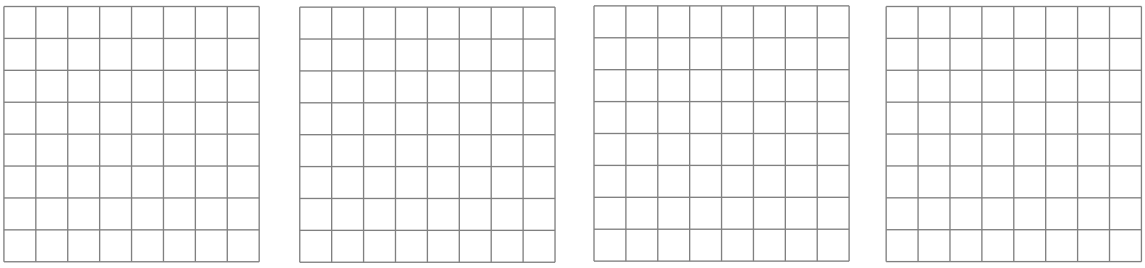

# Practice A

**In Exercises 1–4, simplify the expression.**

**1.** $e^{-9} \cdot e^{12}$ **2.** $\frac{25e^2}{35e^7}$ **3.** $\left(2e^{-3x}\right)^5 \cdot 2e^{x+1}$ **4.** $\sqrt[4]{16e^{24x}}$

**In Exercises 5–8, tell whether the function represents *exponential growth* or *exponential decay.* Then graph the function.**

**5.** $y = 2e^{-x}$ **6.** $y = 0.75e^{4x}$ **7.** $y = 5e^{0.25x}$ **8.** $y = 0.8e^{-3x}$



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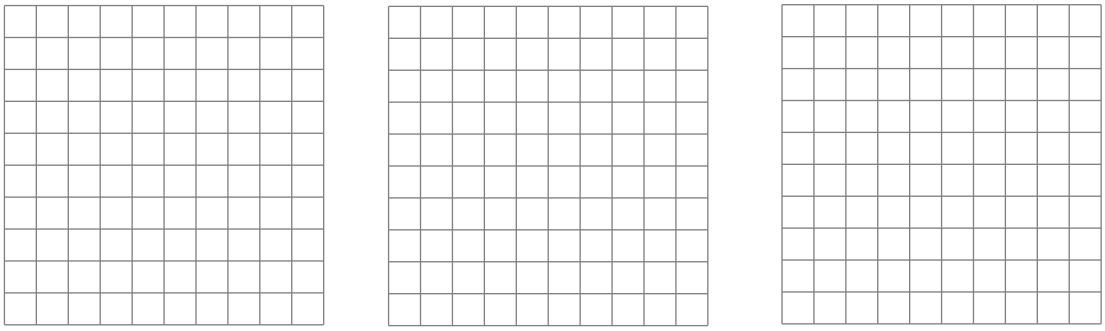
## 3.2 Practice (continued)

**In Exercises 9–11, use a table of values or a graphing calculator to graph the function. Then identify the domain and range.**

**9.** $y = e^x - 4$

**10.** $y = 2e^{x+3}$

**11.** $y = -e^x + 5$



**12.** The population of Evans City is currently 48,500 and is declining at a rate of 2.5% each year. You can model the population of Evans City by the equation $P_t = P_c e^{rt}$, where $P_c$ is the current population, $P_t$ is the population after $t$ years, and $r$ is the decimal rate of decline per year. Predict the population of Evans City after 25 years.

**13.** Your parents will need $25,000 in 10 years to pay for your brother's college tuition. They can invest in an account with an interest rate of 9.8% that compounds continuously. How much should your parents invest today in order to have your brother's full tuition available in 10 years?

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# Practice B

**In Exercises 1–6, simplify the expression.**

**1.** $e^{-9} \cdot e^{7}$

**2.** $\frac{27e^{4}}{18e^{7}}$

**3.** $\left(5e^{-4x}\right)^{3}$

**4.** $\sqrt{20e^{8x}}$

**5.** $\sqrt[3]{64e^{9x}}$

**6.** $e^{2x} \cdot e^{5} \cdot e^{x-2}$

**7.** Describe and correct the error in simplifying the expression.

$$\left(2e^{-3x}\right)^{4} = \frac{1}{16e^{12x}}$$

**In Exercises 8–10, tell whether the function represents *exponential growth* or *exponential decay*. Then graph the function.**

**8.** $y = 2e^{3x}$

**9.** $y = 0.5e^{-2x}$

**10.** $y = 0.4e^{0.5x}$

**In Exercises 11–13, use the properties of exponents to rewrite the function in the form $y = a(1 + r)^t$ or $y = a(1 - r)^t$. Then find the percent rate of change.**

**11.** $y = e^{0.25x}$

**12.** $y = 3e^{-0.65x}$

**13.** $y = 0.25e^{0.9x}$

**In Exercises 14–16, use a table of values or a graphing calculator to graph the function. Then identify the domain and range.**

**14.** $y = e^{x-4}$

**15.** $y = 4e^{x} - 1$

**16.** $y = 2e^{x} + 5$

**17.** You invest $5000 in an account to save for college.

**a.** Option 1 pays 4% annual interest compounded monthly. What would be the balance in the account after 2 years?

**b.** Option 2 pays 4% annual interest compounded continuously. What would be the balance in the account after 2 years?

**c.** What is the difference between the two options after 10 years?

**d.** How would your answer to part (c) change if you invested $50,000?