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3.3 Logarithms and Logarithmic Functions

For use with Exploration 3.3

Essential Question What are some of the characteristics of the graph of a logarithmic function?

Every exponential function of the form $f(x) = b^x$, where b is a positive real number other than 1, has an inverse function that you can denote by $g(x) = \log_b x$. This inverse function is called a *logarithmic function with base b*.

1 EXPLORATION: Rewriting Exponential Equations

Work with a partner. Find the value of x in each exponential equation. Explain your reasoning. Then use the value of x to rewrite the exponential equation in its equivalent logarithmic form, $x = \log_b y$.

a. $2^x = 8$ **b.** $3^x = 9$ **c.** $4^x = 2$

d. $5^x = 1$ **e.** $5^x = \frac{1}{5}$ **f.** $8^x = 4$

2 EXPLORATION: Graphing Exponential and Logarithmic Functions

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Complete each table for the given exponential function. Use the results to complete the table for the given logarithmic function. Explain your reasoning. Then sketch the graphs of f and g in the same coordinate plane.

a.

x	−2	−1	0	1	2
$f(x) = 2^x$					

x					
$g(x) = \log_2 x$	−2	−1	0	1	2

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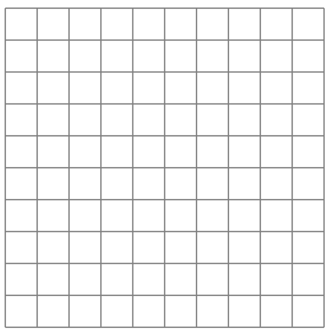
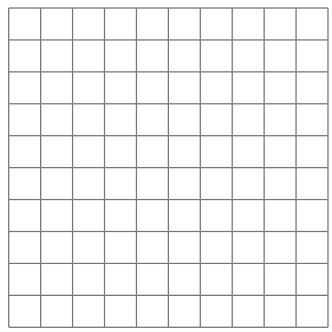
2 EXPLORATION: Graphing Exponential and Logarithmic Functions (continued)

b.

x	-2	-1	0	1	2
$f(x) = 10^x$					

x					
$g(x) = \log_{10} x$	-2	-1	0	1	2





3 EXPLORATION: Characteristics of Graphs of Logarithmic Functions

Work with a partner. Use the graphs you sketched in Exploration 2 to determine the domain, range, x-intercept, and asymptote of the graph of $g(x) = \log_b x$, where b is a positive real number other than 1. Explain your reasoning.

Communicate Your Answer

4. What are some of the characteristics of the graph of a logarithmic function?

5. How can you use the graph of an exponential function to obtain the graph of a logarithmic function?

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3.3 Practice
For use after Lesson 3.3

Core Concepts

Definition of Logarithm with Base *b*

Let b and y be positive real numbers with $b \neq 1$. The **logarithm of y with base b** is denoted by $\log_b y$ and is defined as

$$\log_b y = x \qquad \text{if and only if} \qquad b^x = y.$$

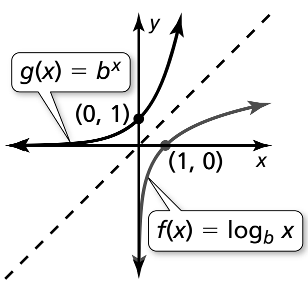
The expression $\log_b y$ is read as "log base b of y."

Notes:

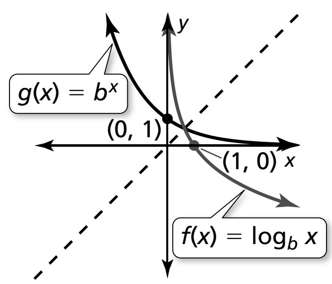
Parent Graphs for Logarithmic Functions

The graph of $f(x) = \log_b x$ is shown below for $b > 1$ and for $0 < b < 1$. Because $f(x) = \log_b x$ and $g(x) = b^x$ are inverse functions, the graph of $f(x) = \log_b x$ is the reflection of the graph of $g(x) = b^x$ in the line $y = x$.

Graph of $f(x) = \log_b x$ for $b > 1$



Graph of $f(x) = \log_b x$ for $0 < b < 1$



Note that the y-axis is a vertical asymptote of the graph of $f(x) = \log_b x$. The domain of $f(x) = \log_b x$ is $x > 0$, and the range is all real numbers.

Notes:

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3.3 **Practice (continued)**

Worked-Out Examples

Example #1

Rewrite the equation in logarithmic form.

$125^{2/3} = 25$, so $\log_{125} 25 = \frac{2}{3}$.

Example #2

Evaluate the logarithm.

$\log_4 0.25 = \log_4 \frac{1}{4} = \log_4 4^{-1} = -1$

Practice A

In Exercises 1–4, rewrite the equation in exponential form.

1. $\log_{10} 1000 = 3$ **2.** $\log_5 \frac{1}{25} = -2$ **3.** $\log_{10} 1 = 0$ **4.** $\log_{1/4} 64 = -3$

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3.3 Practice (continued)

In Exercises 5–8, rewrite the equation in logarithmic form.

5. $12^2 = 144$ **6.** $20^{-1} = \frac{1}{20}$ **7.** $216^{1/3} = 6$ **8.** $4^0 = 1$

In Exercises 9–12, evaluate the logarithm.

9. $\log_4 64$ **10.** $\log_{1/8} 1$ **11.** $\log_2 \frac{1}{32}$ **12.** $\log_{1/25} \frac{1}{5}$

In Exercises 13 and 14, simplify the expression.

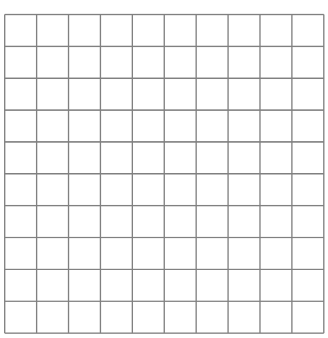
13. $13^{\log_{13} 6}$ **14.** $\ln e^{x^3}$

In Exercises 15 and 16, find the inverse of the function.

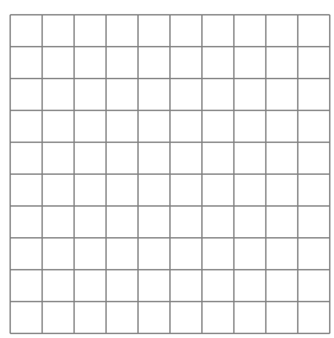
15. $y = 15^x + 10$ **16.** $y = \ln(2x) - 8$

In Exercises 17 and 18, graph the function. Determine the asymptote of the function.

17. $y = \log_2(x + 1)$



18. $y = \log_{1/2} x - 4$



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Practice B

In Exercises 1–3, rewrite the equation in exponential form.

1. $\log_9 1 = 0$ **2.** $\log_6 216 = 3$ **3.** $\log_2 \frac{1}{4} = -2$

In Exercises 4–6, rewrite the equation in logarithmic form.

4. $13^{-2} = \frac{1}{169}$ **5.** $4^{3/2} = 8$ **6.** $81^{1/2} = 9$

In Exercises 7–12, evaluate the logarithm.

7. $\log_8 64$ **8.** $\log_2 32$ **9.** $\log_{10} 1$

10. $\log_3 \frac{1}{81}$ **11.** $\log_2 0.125$ **12.** $\log_{10} 0.01$

In Exercises 13–15, evaluate the logarithm using a calculator. Round your answer to three decimal places.

13. $\log\left(\frac{1}{5}\right)$ **14.** $2\ln(1.4)$ **15.** $\ln(0.4) - 2$

16. The decibel level D of sound is given by the equation $D = 10\log\left(\frac{I}{10^{-12}}\right)$, where I is the intensity of the sound. The pain threshold for sound is 125 decibels. Does a sound with an intensity of 10 exceed the pain threshold? Explain.

In Exercises 17–19, simply the expression.

17. $e^{\ln 7x}$ **18.** $10^{\log 18}$ **19.** $\log\left(10^{3x}\right)$

In Exercises 20–25, find the inverse of the function.

20. $y = 0.75^x$ **21.** $y = \log_{3/4} x$ **22.** $y = \log\left(\frac{x}{2}\right)$

23. $y = \ln(x + 2)$ **24.** $y = e^{x-3}$ **25.** $y = 6^x + 2$

26. The length ℓ (in inches) of an alligator and its weight w (in pounds) are related by the function $\ell = 27.1\ln w - 32.8$.

a. Estimate the length (in inches) of an alligator that weighs 250 pounds. What is its length in feet?

b. Find the inverse of the given function. Use the inverse function to find the weight of a 14-foot alligator. (*Hint*: Convert to inches first.)

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