Name ______________________________ Date __________

3.4 Transformations of Exponential and Logarithmic Functions

For use with Exploration 3.4

Essential Question How can you transform the graphs of exponential and logarithmic functions?

1 EXPLORATION: Identifying Transformations

Work with a partner. Each graph shown is a transformation of the parent function

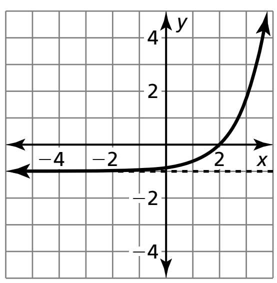
$f(x) = e^x$ or $f(x) = \ln x.$

Match each function with its graph. Explain your reasoning. Then describe the transformation of f represented by g.

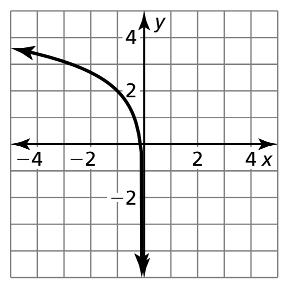
a. $g(x) = e^{x+2} - 3$ **b.** $g(x) = -e^{x+2} + 1$ **c.** $g(x) = e^{x-2} - 1$

d. $g(x) = \ln(x + 2)$ **e.** $g(x) = 2 + \ln x$ **f.** $g(x) = 2 + \ln(-x)$

A.



B.



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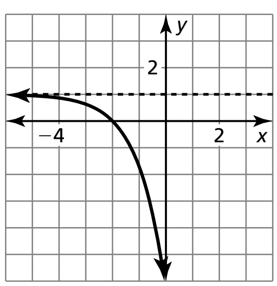

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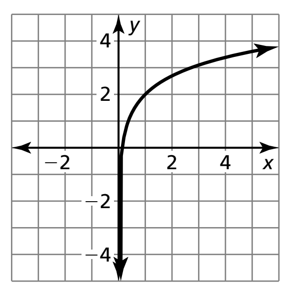
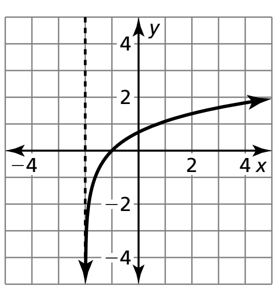
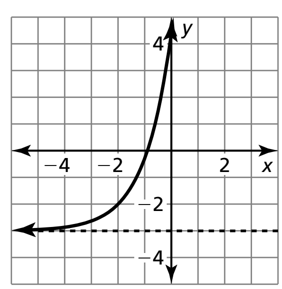


E.



F.



3.4 Transformations of Exponential and Logarithmic Functions (continued)

2 EXPLORATION: Characteristics of Graphs

Work with a partner. Determine the domain, range, and asymptote of each function in Exploration 1. Justify your answers.

Communicate Your Answer

3. How can you transform the graphs of exponential and logarithmic functions?

4. Find the inverse of each function in Exploration 1. Then check your answer by using a graphing calculator to graph each function and its inverse in the same viewing window.

Name ____ Date __________

3.4 Practice

For use after Lesson 3.4

Core Concepts

Transformation	$f(x)$ Notation	Examples	
Horizontal Translation Graph shifts left or right.	$f(x-h)$	$g(x) = 4^{x-3}$ $g(x) = 4^{x+2}$	3 units right 2 units left
Vertical Translation Graph shifts up or down.	$f(x)+k$	$g(x) = 4^x + 5$ $g(x) = 4^x - 1$	5 units up 1 unit down
Reflection Graph flips over x- or y-axis.	$f(-x)$ $-f(x)$	$g(x) = 4^{-x}$ $g(x) = -4^x$	in the y-axis in the x-axis
Horizontal Stretch or Shrink Graph stretches away from or shrinks toward y-axis	$f(ax)$	$g(x) = 4^{2x}$ $g(x) = 4^{x/2}$	shrink by a factor of $\frac{1}{2}$ stretch by a factor of 2
Vertical Stretch or Shrink Graph stretches away from or shrinks toward x-axis	$a \bullet f(x)$	$g(x) = 3(4^x)$ $g(x) = \frac{1}{4}(4^x)$	stretch by a factor of 3 shrink by a factor of $\frac{1}{4}$

Notes:

Name____ Date__________

3.4 Practice (continued)

Transformation	$f(x)$ Notation	Examples	
Horizontal Translation Graph shifts left or right.	$f(x-h)$	$g(x) = \log(x-4)$ $g(x) = \log(x+7)$	4 units right 7 units left
Vertical Translation Graph shifts up or down.	$f(x)+k$	$g(x) = \log x + 3$ $g(x) = \log x - 1$	3 units up 1 unit down
Reflection Graph flips over x- or y-axis.	$f(-x)$ $-f(x)$	$g(x) = \log(-x)$ $g(x) = -\log x$	in the y-axis in the x-axis
Horizontal Stretch or Shrink Graph stretches away from or shrinks toward y-axis	$f(ax)$	$g(x) = \log(4x)$ $g(x) = \log\left(\frac{1}{3}x\right)$	shrink by a factor of $\frac{1}{4}$ stretch by a factor of 3
Vertical Stretch or Shrink Graph stretches away from or shrinks toward x-axis	$a \bullet f(x)$	$g(x) = 5\log x$ $g(x) = \frac{2}{3}\log x$	stretch by a factor of 5 shrink by a factor of $\frac{2}{3}$

Notes:

Worked-Out Examples

Example #1

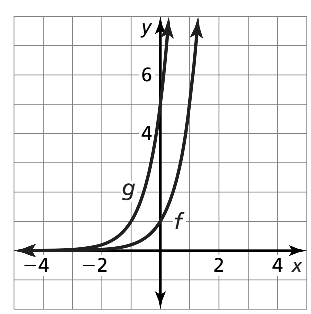
Describe the transformation of f represented by g. Then graph each function.

$f(x) = 5^x$, $g(x) = 5^{x+1}$

Notice that the function is of the form $g(x) = 5^{x-h}$. Rewrite the function to identify h.

$$g(x) = 5^{x-(-1)}$$

Because $h = -1$, the graph of g is a translation 1 unit left of the graph of f.



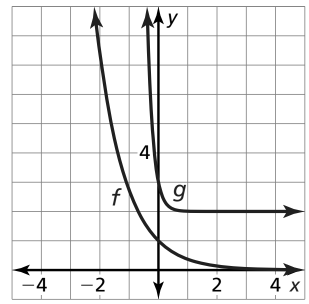
Name ______________________________ Date __________

3.4 Practice (continued)

Example #2

Describe the transformation of f represented by g. Then graph each function.

$f(x) = e^{-x}$, $g(x) = e^{-5x} + 2$

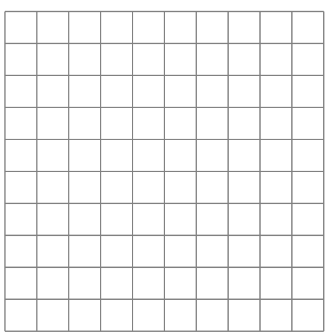


Notice that the function is of the form $g(x) = e^{-ax} + k$, where $a = 5$ and $k = 2$. So, the graph of g is a horizontal shrink by a factor of $\frac{1}{5}$ followed by a translation 2 units up of the graph of f.

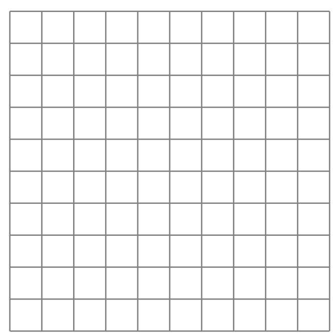
Practice A

In Exercises 1–6, describe the transformation of *f* represented by *g*. Then graph each function.

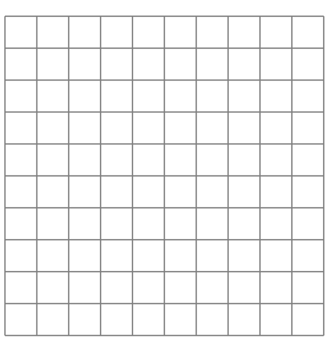
1. $f(x) = 6^x, g(x) = 6^x + 6$



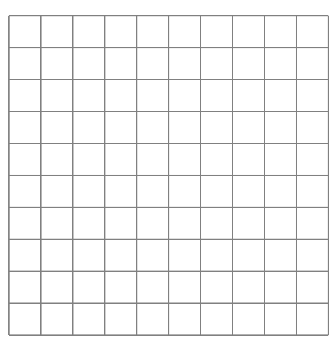
2. $f(x) = e^x, g(x) = e^{x-4}$



3. $f(x) = \log_5 x, g(x) = \frac{1}{2}\log_5(x + 7)$

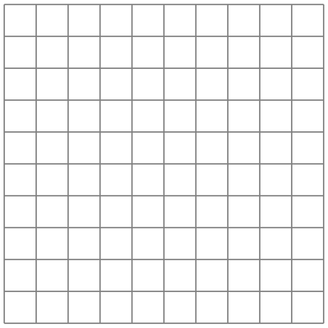


4. $f(x) = \log_{1/3} x, g(x) = \log_{1/3} x - \frac{4}{3}$

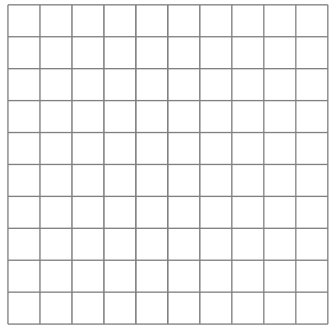


Name____ Date__________

5. $f(x) = \left(\frac{1}{5}\right)^x, g(x) = \left(\frac{1}{5}\right)^{-3x} + 4$



6. $f(x) = \log x, g(x) = -3\log(x - 2)$



Practice B

In Exercises 1–8, describe the transformation of *f* represented by *g*. Then graph each function.

1. $f(x) = e^x, g(x) = e^x - 4$

2. $f(x) = 4^x, g(x) = 4^{x+2}$

3. $f(x) = e^{-x}, g(x) = e^{-x} - 5$

4. $f(x) = \left(\frac{1}{3}\right)^x, g(x) = \left(\frac{1}{3}\right)^x + 2$

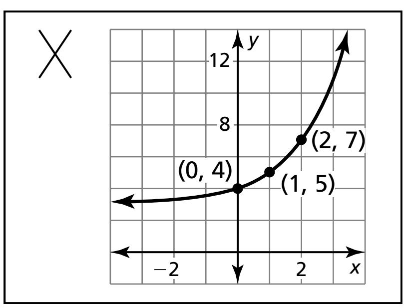
5. $f(x) = 3^x, g(x) = 3^{2x} - 1$

6. $f(x) = e^x, g(x) = -e^{x+2}$

7. $f(x) = e^{-x}, g(x) = e^{-4x+1}$

8. $f(x) = \left(\frac{1}{3}\right)^x, g(x) = \left(\frac{1}{3}\right)^{x-2} + 3$

9. Describe and correct the error in graphing the function $f(x) = 2^{x+3}$.



In Exercises 10 and 11, describe the transformation of *f* represented by *g*. Then graph each function.

10. $f(x) = \log_4 x, g(x) = \log_4(x - 2) + 4$

11. $f(x) = \log_{1/3} x, g(x) = -\log_{1/3}(-x)$

In Exercises 12–14, write a rule for *g* that represents the indicated transformation of the graph of *f*.

12. $f(x) = \left(\frac{2}{5}\right)^x$; reflection in the y-axis, followed by a horizontal shrink by a factor of 2 and a translation 4 units down

13. $f(x) = e^{-x}$; translation 2 units left and 3 units up, followed by a vertical stretch by a factor of 2

14. $f(x) = \log_{12} x$; translation 5 units right and 2 units down, followed by a reflection in the x-axis

Name____ Date__________

3.5 Properties of Logarithms

For use with Exploration 3.5

Essential Question How can you use properties of exponents to derive properties of logarithms?

Let $x = \log_b m$ and $y = \log_b n.$

The corresponding exponential forms of these two equations are

$b^x = m$ and $b^y = n.$

1 EXPLORATION: Product Property of Logarithms

Work with a partner. To derive the Product Property, multiply m and n to obtain

$$mn = b^x b^y = b^{x+y}.$$

The corresponding logarithmic form of $mn = b^{x+y}$ is $\log_b mn = x + y$. So,

$\log_b mn =$ __________. Product Property of Logarithms

2 EXPLORATION: Quotient Property of Logarithms

Work with a partner. To derive the Quotient Property, divide m by n to obtain

$$\frac{m}{n} = \frac{b^x}{b^y} = b^{x-y}.$$

The corresponding logarithmic form of $\frac{m}{n} = b^{x-y}$ is $\log_b \frac{m}{n} = x - y$. So,

$\log_b \frac{m}{n} =$ __________. Quotient Property of Logarithms

3 EXPLORATION: Power Property of Logarithms

Work with a partner. To derive the Power Property, substitute b^x for m in the expression $\log_b m^n$, as follows.

$\log_b m^n = \log_b \left(b^x\right)^n$	Substitute b^x for m.
$= \log_b b^{nx}$	Power of a Power Property of Exponents
$= nx$	Inverse Property of Logarithms

3 EXPLORATION: Power Property of Logarithms (continued)

So, substituting $\log_b m$ for x, you have

$\log_b m^n = $ _________. Power Property of Logarithms

Communicate Your Answer

4. How can you use properties of exponents to derive properties of logarithms?

5. Use the properties of logarithms that you derived in Explorations 1–3 to evaluate each logarithmic expression.

a. $\log_4 16^3$

b. $\log_3 81^{-3}$

c. $\ln e^2 + \ln e^5$

d. $2 \ln e^6 - \ln e^5$

e. $\log_5 75 - \log_5 3$

f. $\log_4 2 + \log_4 32$