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3.5 Properties of Logarithms

For use with Exploration 3.5

Essential Question How can you use properties of exponents to derive properties of logarithms?

Let $x = \log_b m$ and $y = \log_b n$.

The corresponding exponential forms of these two equations are

$b^x = m$ and $b^y = n$.

1 EXPLORATION: Product Property of Logarithms

Work with a partner. To derive the Product Property, multiply m and n to obtain

$$mn = b^x b^y = b^{x+y}.$$

The corresponding logarithmic form of $mn = b^{x+y}$ is $\log_b mn = x + y$. So,

$\log_b mn =$ __________. Product Property of Logarithms

2 EXPLORATION: Quotient Property of Logarithms

Work with a partner. To derive the Quotient Property, divide m by n to obtain

$$\frac{m}{n} = \frac{b^x}{b^y} = b^{x-y}.$$

The corresponding logarithmic form of $\frac{m}{n} = b^{x-y}$ is $\log_b \frac{m}{n} = x - y$. So,

$\log_b \frac{m}{n} =$ __________. Quotient Property of Logarithms

3 EXPLORATION: Power Property of Logarithms

Work with a partner. To derive the Power Property, substitute b^x for m in the expression $\log_b m^n$, as follows.

$\log_b m^n$	$= \log_b \left(b^x\right)^n$	Substitute b^x for m.
	$= \log_b b^{nx}$	Power of a Power Property of Exponents
	$= nx$	Inverse Property of Logarithms

3 EXPLORATION: Power Property of Logarithms (continued)

So, substituting $\log_b m$ for x, you have

$\log_b m^n =$ __________. Power Property of Logarithms

Communicate Your Answer

4. How can you use properties of exponents to derive properties of logarithms?

5. Use the properties of logarithms that you derived in Explorations 1–3 to evaluate each logarithmic expression.

a. $\log_4 16^3$

b. $\log_3 81^{-3}$

c. $\ln e^2 + \ln e^5$

d. $2 \ln e^6 - \ln e^5$

e. $\log_5 75 - \log_5 3$

f. $\log_4 2 + \log_4 32$

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3.5 Practice
For use after Lesson 3.5

Core Concepts

Properties of Logarithms

Let b, m, and n be positive real numbers with $b \neq 1$.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

Notes:

Change-of-Base Formula

If a, b, and c are positive real numbers with $b \neq 1$ and $c \neq 1$, then

$$\log_c a = \frac{\log_b a}{\log_b c}.$$

In particular, $\log_c a = \frac{\log a}{\log c}$ and $\log_c a = \frac{\ln a}{\ln c}$.

Notes:

Worked-Out Examples

Example #1

Expand the logarithmic expression.

$\log_3 4x = \log_3 4 + \log_3 x$

Example #2

Condense the logarithmic expression.

$\log_5 4 + \frac{1}{3}\log_5 x = \log_5 4 + \log_5 \sqrt[3]{x} = \log_5 4\sqrt[3]{x}$

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Practice A

In Exercises 1–4, use $\log_2 5 \approx 2.322$ and $\log_2 12 \approx 3.585$ to evaluate the logarithm.

1. $\log_2 60$

2. $\log_2 \frac{1}{144}$

3. $\log_2 \frac{12}{25}$

4. $\log_2 720$

In Exercises 5–8, expand the logarithmic expression.

5. $\log 10x$

6. $\ln 2x^6$

7. $\log_3 \frac{x^4}{3y^3}$

8. $\ln \sqrt[4]{3y^2}$

In Exercises 9–13, condense the logarithmic expression.

9. $\log_2 3 + \log_2 8$

10. $\log_5 4 - 2\log_5 5$

11. $3\ln 6x + \ln 4y$

12. $\log_2 625 - \log_2 125 + \frac{1}{3}\log_2 27$

13. $-\log_6 6 - \log_6 2y + 2\log_6 3x$

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3.5 Practice (continued)

In Exercises 14–17, use the change-of-base formula to evaluate the logarithm.

14. $\log_3 17$ **15.** $\log_9 294$ **16.** $\log_7 \frac{4}{9}$ **17.** $\log_6 \frac{1}{10}$

18. For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function $L(I) = 10 \log \frac{I}{I_0}$, where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). The intensity of the sound of a certain children's television show is half the intensity of the adult show that is on before it. By how many decibels does the loudness decrease?

19. Hick's Law states that given n equally probable choices, such as choices on a menu, the average human's reaction time T (in seconds) required to choose from those choices is approximately $T = a + b \bullet \log_2(n + 1)$ where a and b are constants. If $a = 4$ and $b = 1$, how much longer would it take a customer to choose what to eat from a menu of 40 items than from a menu of 10 items?

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Practice B

In Exercises 1–3, use $\log_5 3 \approx 0.683$ and $\log_5 6 \approx 1.113$ to evaluate the logarithm.

1. $\log_5 81$
2. $\log_5 \frac{1}{6}$
3. $\log_5 \frac{1}{2}$

In Exercises 4–6, expand the logarithmic expression.

4. $\log_3 12x^7$
5. $\log_6 \dfrac{5x^2}{y^3}$
6. $\log_8 6\sqrt{xy}$

7. Describe and correct the error in expanding the logarithmic expression.

$$\times \quad \ln \sqrt[3]{xy} = \frac{1}{3} \ln x + \ln y$$

In Exercises 8–11, condense the logarithmic expression.

8. $5 \log_9 x - \log_9 4$
9. $\log_8 5 + \frac{1}{4} \log_8 x$
10. $2 \ln 4 + 5 \ln x + 3 \ln y$
11. $\log_6 9 + 2 \log_6 \frac{1}{3} - 3 \log_6 x$

In Exercises 12–14, use the change-of-base formula to evaluate the logarithm.

12. $\log_8 15$
13. $\log_3 30$
14. $\log_4 \frac{8}{17}$

15. Your friend claims you can use the change-of-base formula to write the expression $\dfrac{\ln y}{\ln 3}$ as a logarithm with base 3. Is your friend correct? Explain your reasoning.

16. For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function $L(I) = 10 \log \dfrac{I}{I_0}$, where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). The bass guitar player in a band turns up the volume of the speaker so that the intensity of the sound triples. By how many decibels does the loudness increase?